A UNIFIED FIELD THEORY * MILES V. HAYES

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by

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Dedicated to the memory of

ALBERT EINSTEIN

EINSTEIN concluded that physical reality must be described by fields (gravitational, electromagnetic, or others), that is by clearly defined magnitudes at every point of space and time. To his mind, the particle must be defined not as a field source (the usual way of defining it in electron or gravitational theory) but as a very small region of space in which the field assumes very high, though finite, values. The properties and motions of particles then would be completely given by field equations.

'There will be no place in the new physics,' Einstein wrote, 'for both fields and matter, that is, particles, because fields will be the only reality.'

from LOUIS DE BROGLIE, *Industrial Research*

October, 1962

Summary

THE universe consists of a complex quaternionic field which is a function of space-time such that, in the sense defined by the algebra of complex quaternions, its rate of change is proportional to the square of its magnitude.

Stated mathematically, reality is determined by the partial differential field equation

$$D\psi = \tfrac{1}{2}\psi^{*}\iota\psi$$

together with initial and boundary values of ψ for any finite region of space-time, where

$$D = \iota\frac{\partial}{\partial t} + \nabla = \iota\frac{\partial}{\partial t} + i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\psi = H + \iota E = iH_x + jH_y + kH_z + i\iota E_x + j\iota E_y + k\iota E_z$$

$$\psi^{*} = H - \iota E = iH_x + jH_y + kH_z - i\iota E_x - j\iota E_y - k\iota E_z$$

and the algebra of complex quaternions is defined by the relations

$\iota\iota = ii = jj = kk = -1$ (ι=iota, i=eye, 1=one)

$\iota i = i\iota$, $\iota j = j\iota$, $\iota k = k\iota$

$ij = -ji = k$, $jk = -kj = i$, $ki = -ik = j$

ι is the ordinary imaginary unit and i, j, and k are the ordinary unit vectors, as defined by Hamilton in his quaternions.

Using the algebra, the field equation can be expanded to

$$\left(\iota\frac{\partial}{\partial t} + \nabla\right)(H + \iota E) = \tfrac{1}{2}(H - \iota E)\,\iota(H + \iota E)$$

or

$$-\nabla\cdot H - \iota\nabla\cdot E + \left(\nabla\times H - \frac{\partial E}{\partial t}\right) + \iota\left(\nabla\times E + \frac{\partial H}{\partial t}\right) = -\tfrac{1}{2}\iota(H^2 + E^2) + E\times H$$

The field equation is expressed in the natural system of units obtained by selecting new units of say time and mass so that the velocity of light c=1 and the ratio of charge to mass of the electron e/m=1. In such a system of units energy density ε, mass density

m, and electric charge density ρ are identical and are all equal to $\frac{1}{2}(H^2+E^2)$; while energy flux density εv, momentum density mv, and electric current density ρv are identical and are all equal to $E\times H$. The sum of the two, $-\iota\frac{1}{2}(H^2+E^2)+E\times H$, is the matter quaternion.

The probability amplitude vector ψ is a complex three-dimensional vector identical with the complex electromagnetic vector $H+\iota E$, except perhaps for a normalization constant. Electromagnetic waves or photons are travelling ψ-waves. Particles having rest mass are standing ψ-waves or wave-packets. Moving particles having rest mass are mixtures of standing and travelling waves. Mechanical motion does not exist at the fundamental field level where the only physical reality is the ψ field vector: the ψ-vector at a point cannot move to any other point. It can only change the magnitudes of its components. All that moves are the maxima and minima, the crests and troughs, of ψ-waves. Particles are propagated as waves, not transported as corpuscles.

Point particles do not exist at the fundamental field level. Particles are ψ-waves which extend over finite volumes. The ψ-waves may define more or less precisely a center of gravity, or center of geometry, or center of interaction, which as a mental convenience may be thought of as a point particle. The positions and velocities of such fictitious point particles can only be defined probabilistically. For, although the ψ-field is causal and determinate over the universe as a whole, as governed by the field equation, the ψ-field is non-linear and so cannot be decomposed into partial or constituent fields by the principle of superposition, except approximately. Any local or partial ψ-field, such as a single particle, is dependent on the ψ-field of the rest of the universe, at least within the relativistic light-cone. The ψ-field of the universe is unknown, and cannot be measured without being changed. Lacking knowledge of the complete universe, one lacks complete knowledge of a single particle, and so must resort to probabilities. The probability density that a point particle, namely the center of geometry of a ψ-wave constituting a particle, will be found in an element of volume is assumed to be proportional to the scalar of $\frac{1}{2}\psi^*\iota\psi$, or $\frac{1}{2}(H^2+E^2)$, which is the energy-mass-charge density. The probability current density is assumed to be proportional to the vector of $\frac{1}{2}\psi^*\iota\psi$, or

$E\times H$, which is the energy flux-momentum-electric current density. The matter quaternion $\frac{1}{2}\psi^*\iota\psi$ is therefore also the point particle probability quaternion.

If one makes the approximation that $\frac{1}{2}\psi^*\iota\psi$ is a constant quaternion C, equal to $-\iota\rho+\rho v$, and independent of ψ though still varying with x, y, z, and t, the field equation becomes Maxwell's equations $D\psi=-\iota\rho+\rho v$ or $D\psi=C$. The field theory accounts for classical electromagnetism, and for its failure in the atomic and nuclear domain where the constant approximation is invalid.

If one makes the better approximation that $\frac{1}{2}\psi^*\iota$ is a constant K independent of ψ, the field equation becomes Dirac's equation $D\psi=K\psi$.

Like Maxwell's and Dirac's equations, which approximate it, the field equation is relativistically invariant in form under a Lorentz transformation, which is true of very few other equations and which is necessary experimentally if an equation is to be true. The field theory gives a good qualitative explanation of special relativity: for example, the limiting speed of a particle is the speed of light because a particle *is* light.

The field equation is a succinct statement of the fundamental postulate of quantum mechanics. If $D\psi=\frac{1}{2}\psi^*\iota\psi$ is written as $(\iota\partial/\partial t+\nabla)\psi=-\iota\frac{1}{2}(H^2+E^2)+E\times H=$ scalar energy + vector momentum, the equation can be read as the statement that scalar energy and vector momentum can be replaced by the operators $\iota\partial/\partial t$ and ∇ operating on the wave function ψ. This postulate, plus the linear eigenvalue form of equation like Dirac's $D\psi=K\psi$, which is the linear approximation to the field equation, accounts for Schroedinger's equation and quantum mechanics, and for the failure of quantum mechanics in the domain of the elementary particles, where the linear approximation is invalid.

Newton's laws of motion and the conservation laws are implicit in the field equation and can be derived from it, so it accounts for classical mechanics.

The field equation identifies electric charge density with electromagnetic field energy density. It relates the sign of electric charge to the right- or left-handedness of the ψ-wave geometries of the elementary particles. Reversing the sign of ψ, which changes its handedness, changes the sign of the linear term $D\psi$ but not the

sign of the quadratic term $\frac{1}{2}\psi^*\iota\psi$, or changes the relative sign of ρ with respect to $D\psi$ in Maxwell's equations. Thus the field theory gives a simple explanation for the existence of electric charge, the two signs of electric charge, the relation of parity or handedness to the sign of charge, the existence of matter and anti-matter as right- and left-handed matter, the opposite signs of charge of matter and anti-matter, and the existence of neutral and partly neutral particles like neutron and proton as partly right-handed and partly left-handed distributions of ψ.

The discrete properties of matter, including the fact that electric charge occurs as integral multiples of the charge of the electron, that matter occurs as multiples of elementary particles, that radiation is emitted and absorbed in quanta, and that the Uncertainty Principle demands discrete Fourier sums instead of continuous Fourier integrals, are attributed to the fact that the field equation is non-linear and exhibits the typical characteristic of non-linear differential equations of having oscillatory solutions at discrete stable limit cycles, rather than a continuum of solutions depending on the initial conditions as with linear equations. The field itself however is continuous.

The field equation, with its initial and boundary conditions, is self-sufficient and so does not need and *cannot tolerate* any independent equations. The field vector ψ is the only physical reality. All fields, whether electromagnetic, probability amplitude, gravitational, or nuclear, are ψ-fields. All energy, charge, mass, and probability densities, which are $\frac{1}{2}(H^2+E^2)$, and all energy flux, electric current, momentum, and probability current densities, which are $E\times H$, are components of the matter quaternion $\frac{1}{2}\psi^*\iota\psi$. All elementary particles, atoms, molecules, and macroscopic bodies are standing or travelling ψ-waves, of various more or less elaborate geometrical descriptions. All laws, equations, and constants of physics, chemistry, and biology are implicit in and to be derived from the field equation. The field equation is the fundamental and the only fundamental law of existence.

The unified field theory reveals Nature to be elegantly and mysteriously simple.

Contents

Viewpoint

THE work I commence is a description of the physical and metaphysical foundations of an engineer's philosophy. My viewpoint is primarily philosophical and secondarily physical. I want to see life whole and in proportion. I wish to coordinate in a single theory, as best I can, the demonstrable and observable facts of science, and, where demonstrable facts are unavailable as in the realm of mind and value, the working hypotheses or articles of faith that seem to me most plausible.

With these thoughts in mind I sat down in December of 1961 to start a book on an engineer's philosophy with a discussion of physics and metaphysics, roughed in with very broad brush strokes, and of a qualitative rather than a quantitative nature. I resolved to be unmindful of mathematical difficulties and niceties in order to lunge as far as I could. To my utter astonishment, although I had only expected qualitative generalities, the beginnings of a precise theory formed in my mind which within forty-eight hours I wrote down as a set of physical hypotheses. One of them was the field equation in embryonic but recognizable mathematical form.

In the following year I developed the theory and submitted three papers to the Physical Review. They were rejected on the grounds that the theory is 'speculative'. Had all scientists refrained from speculation there would be no physics and no Physical Review. I have decided to publish the theory privately.

The difficulty in gaining any acceptance or even consideration for the theory is not, I think, that it fails to explain physics but that it explains too much too simply. It seems incredible that a short simple equation $D\psi = \frac{1}{2}\psi^* \iota \psi$ should be adequate to explain all of physical reality. I don't blame physicists for being skeptical, since I have a great deal of trouble believing it myself.

But I have even more trouble believing the theory is false when it explains so much. What other theory is proposed that better explains, or indeed explains at all, the existence of classical electromagnetism, classical mechanics, quantum mechanics, electric charge, matter and anti-matter, special relativity, the conservation laws, and promises to explain the elementary particles? I am open to conversion, but until someone shows that the theory leads to fallacious results, which no one has done, or someone produces a simpler and broader theory, which no one has done, I propose to accept on a tentative basis the unified field theory, and I recommend that others at least consider doing likewise.

The Field Equation

It is postulated that reality is completely and exactly described and determined, at all times and places and at any scale large or small, by the single, self-sufficient, quaternionic, partial differential field equation

$$D\psi = \tfrac{1}{2}\psi^* \iota \psi$$

together with initial and boundary values of ψ for any finite region, where

$$D = \iota\frac{\partial}{\partial t} + \nabla = \iota\frac{\partial}{\partial t} + i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\psi = H + \iota E = iH_x + jH_y + kH_z + i\iota E_x + j\iota E_y + k\iota E_z$$

$$\psi^* = H - \iota E = iH_x + jH_y + kH_z - i\iota E_x - j\iota E_y - k\iota E_z$$

and the algebra of complex quaternions is defined by the relations

$\iota = ii = jj = kk = -1$ (ι = iota, i = eye, 1 = one)

$i\iota = i\iota$, $j\iota = j\iota$, $k\iota = k\iota$

$ij = -ji = k$, $jk = -kj = i$, $ki = -ik = j$

ι is the ordinary commutative imaginary unit of two-dimensional complex numbers and is the square root of minus one.

i, j, and k are the ordinary non-commutative unit vectors of

three-dimensional space, as originally defined by Hamilton in his quaternions, where they too are square roots of minus one.

A real quaternion consists of the sum of a real scalar plus a three-dimensional real vector. It is a four-dimensional hypercomplex number. An imaginary quaternion consists of a real quaternion multiplied by the unit imaginary ι. A complex quaternion is the sum of a real quaternion and an imaginary quaternion. In general a complex quaternion occupies an eight-dimensional space, with units 1, ι, i, j, k, $i\iota$, $j\iota$, and $k\iota$ along the real scalar, imaginary scalar, three real vector, and three imaginary vector axes respectively. The field quaternion $D\psi$ occupies all eight. The field vector ψ is scalarless and occupies six. Space-time, a typical interval of which is $\iota t + ix + jy + kz$; the quaternionic rate-of-change operator $D = \iota\partial/\partial t + i\partial/\partial x + j\partial/\partial y + k\partial/\partial z$; and the matter quaternion $\frac{1}{2}\psi^*\iota\psi = -\frac{1}{2}\iota(H^2+E^2) + E\times H$ occupy four, namely ι, i, j, and k, which are the four of the eight equalling the square root of minus one.

Application of the algebra of quaternions gives the quaternionic product of two vectors A and B to be

$$\begin{aligned} AB &= (iA_x + jA_y + kA_z)\ (iB_x + jB_y + kB_z) \\ &= ii A_xB_x + jj A_yB_y + kk A_zB_z + jk A_yB_z + kj A_zB_y \\ &\quad + ki A_zB_x + ik A_xB_z + ij A_xB_y + ji A_yB_x \\ &= -(A_xB_x + A_yB_y + A_zB_z) + i(A_yB_z - A_zB_y) \\ &\quad + j(A_zB_x - A_xB_z) + k(A_xB_y - A_yB_x) \\ &= -A\cdot B + A\times B \end{aligned}$$

So the quaternionic product of two vectors is the negative of their dot or scalar product plus their cross or vector product. In particular, if $A = \nabla = i\partial/\partial x + j\partial/\partial y + k\partial/\partial z$ then

$$\nabla B = -\nabla\cdot B + \nabla\times B$$

Using this relation to expand the field equation

$$D\psi = \tfrac{1}{2}\psi^*\iota\psi$$

$$\left(\iota\frac{\partial}{\partial t} + \nabla\right)(H + \iota E) = \tfrac{1}{2}(H - \iota E)\iota(H + \iota E)$$

$$-\nabla\cdot H - \iota\nabla\cdot E + \left(\nabla\times H - \frac{\partial E}{\partial t}\right) + \iota\left(\nabla\times E + \frac{\partial H}{\partial t}\right) = \tfrac{1}{2}\iota(-H\cdot H + H\times H - E\cdot E + E\times E - \iota H\cdot E + \iota H\times E + \iota E\cdot H - \iota E\times H)$$

$$= -\tfrac{1}{2}\iota(H^2+E^2) + E\times H$$

Separating real and imaginary scalars and vectors, the field equation becomes four equations:

Real scalar: $-\nabla\cdot H=0$

Imaginary scalar: $-\iota\nabla\cdot E=-\frac{1}{2}\iota(H^2+E^2)$

Real vector: $\nabla\times H-\partial E/\partial t=E\times H$

Imaginary vector: $\iota\nabla\times E+\iota\partial H/\partial t=0$

Expanding each of the vector equations into its three components, the field equation becomes eight equations, as follows:

Axis

1: $$-\frac{\partial H_x}{\partial x}-\frac{\partial H_y}{\partial y}-\frac{\partial H_z}{\partial z}=0$$

ι: $$-\frac{\partial E_x}{\partial x}-\frac{\partial E_y}{\partial y}-\frac{\partial E_z}{\partial z}=-\frac{1}{2}(E_x^2+E_y^2+E_z^2+H_x^2+H_y^2+H_z^2)$$

i: $$\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}-\frac{\partial E_x}{\partial t}=E_yH_z-E_zH_y$$

j: $$\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x}-\frac{\partial E_y}{\partial t}=E_zH_x-E_xH_z$$

k: $$\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}-\frac{\partial E_z}{\partial t}=E_xH_y-E_yH_x$$

ιi: $$\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}+\frac{\partial H_x}{\partial t}=0$$

ιj: $$\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}+\frac{\partial H_y}{\partial t}=0$$

ιk: $$\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}+\frac{\partial H_z}{\partial t}=0$$

These eight equations, with initial and boundary conditions, determine H_x, H_y, H_z, E_x, E_y, and E_z as functions of x, y, z, and t. It might appear that eight equations overdetermine six unknowns, but there are really eight unknowns since in general H and E might be quaternions with scalar parts instead of being three-vectors as they are, so the eight equations may be looked upon as six determining the six vector components of H and E, and two ensuring that the scalars of H and E shall be zero. They are analogous to the eight equations which constitute Maxwell's equations for free space and which, with initial and boundary conditions, determine the six vector components of H and E, except that here the right-hand sides are quadratic functions of H and E instead of being zero.

ψ is the complex electromagnetic and probability amplitude three-vector, or simply the field vector, consisting of the real magnetic field intensity vector H and the imaginary electric field intensity vector E. The complex electromagnetic vector is identical with the probability amplitude vector, except perhaps for a normalization constant. At the fundamental field level the field vector ψ is the only physical reality. All more elaborate physical entities, such as elementary particles, nuclei, atoms, molecules, cells, organisms, and other macroscopic bodies are geometrical distributions and organizations of the field vector ψ over finite volumes of space and are to be defined in terms of ψ.

D is the quaternionic differential or rate-of-change operator consisting of the imaginary scalar $\iota\partial/\partial t$ and the real vector ∇.

$D\psi$ is the field quaternion or the Maxwell free-space quaternion. It is a full eight-dimensional complex quaternion consisting of the real scalar $-\nabla\cdot H$, the imaginary scalar $-\iota\nabla\cdot E$, the real vector $\nabla\times H-\frac{\partial E}{\partial t}$, and the imaginary vector $\iota\nabla\times E+\iota\frac{\partial H}{\partial t}$. Maxwell's equations for free space are $D\psi=0$.

$\frac{1}{2}\psi^*\iota\psi$ is the matter quaternion, consisting of the imaginary scalar $-\frac{1}{2}\iota(H^2+E^2)$, which is the field energy density, electric charge density, mass density, and point particle probability density, all of which are identical; and the real vector $E\times H$, which is the field energy flux density, electric current density, momentum density, and point particle probability current density, all of which are identical.

The field equation is expressed in the natural system of units obtained by selecting the size of two of the three fundamental units of measurement of length, time, and mass, so that the velocity of light $c=1$ and the ratio of charge to mass of the electron $e/m=1$. In such a system of units energy, charge, and mass are identical. The size of the third fundamental unit is determined by the solutions of the field equation.

The field equation $D\psi=\frac{1}{2}\psi^*\iota\psi$ states that the source of the field is the field itself. It states that the linear field $D\psi$, which is the one we are accustomed to in electromagnetism, wave mechanics, and gravitation, has its source in the quadratic field $\frac{1}{2}\psi^*\iota\psi$, which defines energy, charge, mass, and momentum, and is what we mean by matter. The field has its source in matter, and matter consists of the field.

The field equation $D\psi=\frac{1}{2}\psi^*\iota\psi$ states that reality consists of

a complex quaternionic field which is a function of space-time such that, in the sense defined by the algebra of complex quaternions, its rate of change is proportional to the square of its magnitude.

The Zero-Order Approximation: Maxwell's Equations for Free Space, and an Hypothesis that the Universe is not Expanding

If one assumes that the matter quaternion $\frac{1}{2}\psi^*\iota\psi$ is zero, the field equation becomes $D\psi=0$, which is Maxwell's equations for free space, or

$$-\nabla\cdot H-\iota\nabla\cdot E+(\nabla\times H-\frac{\partial E}{\partial t})+\iota(\nabla\times E+\frac{\partial H}{\partial t})=0$$

Solutions are well known, and consist of travelling electromagnetic or ψ-waves, which propagate with the speed of light. The field theory therefore accounts for the existence of classical light waves.

There is an important difference however between the light waves described by classical electromagnetic theory and light waves described by the unified field theory. In classical theory $D\psi=0$ was assumed to be exactly correct no matter how large ψ was, provided that no charged particles were present, i.e. provided $\rho=0$. In the field theory $D\psi=0$ is exactly correct only in the trivial or limiting case where $\psi=0$. For small ψ, the quadratic terms in $\frac{1}{2}\psi^*\iota\psi$ may be negligible compared to the linear terms in $D\psi$, but, strictly speaking, if $\frac{1}{2}\psi^*\iota\psi=0$ then $\psi=0$, and if ψ is non-zero then $\frac{1}{2}\psi^*\iota\psi$ is non-zero. The equation $D\psi=0$ is never quite accurate.

In classical electromagnetic theory and also in the linear differential equations of quantum mechanics light waves or ψ-waves do not interact.

A system is said to be linear if the effect of two causes working together is the same as the sum of the effects of the same two causes working separately, or $E(C_1+C_2)=E(C_1)+E(C_2)$. A mathematical

operator like D is linear because $D(\psi_1+\psi_2)=D(\psi_1)+D(\psi_2)$, but an operator like $D-\frac{1}{2}\psi^*\iota$ is non-linear because, with $\psi=\psi_1+\psi_2$, it gives $(D-\frac{1}{2}(\psi_1+\psi_2)^*\iota)(\psi_1+\psi_2)\neq(D-\frac{1}{2}\psi_1^*\iota)\psi_1+(D-\frac{1}{2}\psi_2^*\iota)\psi_2$ since the left side contains the cross-product or interaction terms $\frac{1}{2}\psi_1^*\iota\psi_2$ and $\frac{1}{2}\psi_2^*\iota\psi_1$ which are missing on the right. In a linear system two causes or functions or ψ-waves don't know or care whether they exist together or separately. In a non-linear system they do: they interact and modify each other.

Two waves ψ_1 and ψ_2, each of which separately satisfies Maxwell's equation, $D\psi_1=0$ and $D\psi_2=0$, add up to give a combined wave which also satisfies Maxwell's equation, $D(\psi_1+\psi_2)=0$. In the non-linear field theory however the linear equation $D\psi=0$ is never quite correct and must be replaced by $D\psi=\frac{1}{2}\psi^*\iota\psi$, according to which light waves or ψ-waves do interact. If two waves ψ_1 and ψ_2 are separately solutions of the field equation, $D\psi_1=\frac{1}{2}\psi_1^*\iota\psi_1$ and $D\psi_2=\frac{1}{2}\psi_2^*\iota\psi_2$, then in general their sum is *not* a solution of the field equation $D(\psi_1+\psi_2)\neq\frac{1}{2}(\psi_1+\psi_2)^*\iota(\psi_1+\psi_2)$ due to the non-linear interaction terms $\frac{1}{2}\psi_1^*\iota\psi_2$ and $\frac{1}{2}\psi_2^*\iota\psi_1$. Both ψ_1 and ψ_2, if present simultaneously, will interact or be modified somewhat so that their sum *is* a solution of the field equation.

The non-linear interaction of light waves, or the modification of one by another, suggests an explanation for the astronomical red shift.

The red shift is the shift of the frequency of identifiable spectral lines, in the light coming from the stars, to a lower frequency than that produced by the same element on earth. Currently the shift is attributed to the Doppler effect, which in turn is due to the velocity of the source with respect to the observer. The implication is that distant stars are receding from the earth, and the more distant they are the greater the velocity of recession. Hence the expanding universe.

At least one physicist, Alastair Ward, of the Royal College of Science and Technology, Glasgow, has suggested that the Doppler effect is not the correct explanation and that the universe is not expanding. He suggests that photons do interact with photons, in spite of linear theories, and he suggests experiments to test his hypothesis.

The unified field theory offers a theoretical justification for

Ward's hypothesis. The theory predicts that light waves interact with light waves, or photons with photons. The frequency, amplitude, phase, and other characteristics of light waves are modified by interaction with the other light waves through which they pass in coming from the stars. The greater the distance travelled the greater the modification.

Although of course the details of a quantitative theory remain to be worked out and verified, the objections to the hypothesis that the universe is expanding are so great that I tentatively adopt the alternate hypothesis that the red shift is due to the non-linear interaction of light waves and not to the Doppler effect, and that the universe is not expanding.

The Constant Approximation: Maxwell's Equations with Charged Particles

If one assumes that the matter quaternion $\frac{1}{2}\psi^*\iota\psi$ is a constant, independent of ψ, although varying with x, y, z, and t, and lets this constant be $\frac{1}{2}\psi^*\iota\psi = C = -\iota\rho + \rho v$, then the field equation becomes $D\psi = C$, which is Maxwell's equations in the presence of charged particles, and can be written as

$$-\nabla\cdot H - \iota\nabla\cdot E + (\nabla\times H - \frac{\partial E}{\partial t}) + \iota(\nabla\times E + \frac{\partial H}{\partial t}) = -\iota\rho + \rho v$$

Place a fictitious control surface, in the form of a sphere, around each small volume of the field where very intense and variable values of ψ exist, which is to say where any elementary particle having rest mass exists, and assume that the field density inside is a constant. The fictitious sphere constitutes a classical particle, the billiard-ball particle of old-fashioned mechanics. Its center constitutes a point particle. The spherical control surfaces divide physics into three realms:

1) The realm of classical electromagnetism, which is the realm

outside the control surfaces. In this realm the constant approximation $D\psi = C$ is reasonably accurate, so Maxwell's equations apply and govern.

2) The realm of quantum and nuclear physics, which is the realm inside the control surfaces. Here $D\psi = C$ is not a good approximation since $\frac{1}{2}\psi^*\iota\psi$ is neither constant nor small, so Maxwell's equations do not apply and a better approximation to the field equation, such as Dirac's equation $D\psi = K\psi$ or the related Schroedinger's equation must be used, or, better yet, the exact field equation $D\psi = \frac{1}{2}\psi^*\iota\psi$.

3) The realm of classical mechanics, which is the realm of the control surfaces themselves. An independent set of laws, namely Newton's laws of motion and gravitation must be found to govern the control surfaces or classical particles, and must be related to Maxwell's equations by the electromagnetic force equation $F = \rho(E + (1/c)v\times H)$.

The exact field equation $D\psi = \frac{1}{2}\psi^*\iota\psi$ dispenses with the classical particle control surfaces and hence with Newton's laws of motion and gravitation as explicit or independent laws of Nature. Newton's laws are implicit in the field equation and are to be derived from it. The exact field equation unifies the realms inside and outside the control surfaces and gives one governing equation which applies to any field, macroscopic or microscopic.

To summarize, the field equation accounts for the existence of Maxwell's equations as the constant approximation to the field equation. It accounts for the existence of the three apparently separate realms of physics, namely classical electromagnetism, classical mechanics, and atomic and nuclear physics. It accounts for the failure of Maxwell's equations in the atomic and nuclear domain where the constant approximation is invalid. The exact field equation eliminates the need for classical mechanics, and unifies the hitherto separate domains of classical electromagnetism and of atomic and nuclear physics.

Electric Charge, Parity, Matter and Anti-Matter; and an Hypothesis that Matter and Anti-Matter are Equally Abundant

THE first victory of the unified field theory was the explanation of the two signs of electric charge. When I was first led to the theory by a desire to non-linearize Maxwell's equations, I conjectured that electric charge density ρ should be some non-linear function of the field vectors E and H, such as the field energy density $\frac{1}{2}(H^2+E^2)$, but for a while I dismissed the possibility that $\rho=\frac{1}{2}(H^2+E^2)$. It seemed hopeless to equate charge, which can be positive or negative, to energy, which is essentially positive. Then a light dawned on me as I realized that not only could charge and energy appear to be negative as well as positive, but the theory gave a beautifully simple explanation of why particles of matter and anti-matter, like electron and positron, exist, and why they have opposite charges.

The field equation $D\psi=\frac{1}{2}\psi^*\iota\psi$ equates $D\psi$, a term linear in ψ, to $\frac{1}{2}\psi^*\iota\psi$, a quadratic term. If ψ is replaced by $-\psi$ then $+D\psi$ becomes $-D\psi$ but $+\frac{1}{2}\psi^*\iota\psi$ remains $+\frac{1}{2}\psi^*\iota\psi$, giving $-D\psi=+\frac{1}{2}\psi^*\iota\psi$ or $+D\psi=-\frac{1}{2}\psi^*\iota\psi$. So changing the sign of ψ gives the appearance of changing the sign of the matter quaternion $\frac{1}{2}\psi^*\iota\psi$ in the field equation or the sign of ρ in Maxwell's equations, although the matter quaternion and ρ are essentially positive. But what does changing the sign of ψ mean? Since $\psi=H+\iota E$ it means directing the vectors H and ιE oppositely in three-dimensional space. This has the effect of changing right-handed field distributions into left-handed ones, and vice versa. For example suppose one stands with one's head in the direction of an H vector, looks in the direction of a constant ιE vector, and has in the direction of one's right hand a time-varying ιE vector. If all vectors are reversed and one again orients oneself by standing along H looking in the constant ιE direction, the time-varying ιE is now on one's left.

If an electron is an oscillating ψ-wave packet having a certain asymmetry or handedness, say for example right-handedness, then a similar ψ-wave packet, differing only in being left-handed, is a

positron. They differ only in the sign of ψ and therefore really differ in the sign of $D\psi$, so they appear to differ in the sign of $\frac{1}{2}\psi^*\iota\psi$ or in the sign of their charges ρ. The field equation accounts for the existence of particles of matter and anti-matter as ψ-wave distributions having right- and left-handed asymmetry, and accounts for the fact that such particles appear to have opposite signs of charge.

The fact that the field equation accounts for particles of matter and anti-matter is a solid indication that it is valid in the domain of the elementary particles, and since it accounts also for Maxwell's equations and the classical domain, that it is universally valid.

Handedness is usually discussed in physics under the heading of parity. The parity P of a function f(x,y,z) is +1 (even) or −1 (odd) depending on the behavior of the equation

$$f(x,y,z)=Pf(-x,-y,-z)$$

A stir was recently caused in physics by the discovery that parity is not conserved. Charles J. Goebel, in the McGraw-Hill *Encyclopaedia of Science and Technology*, under 'Parity', writes:

'It was at first somewhat disconcerting to find parity not conserved, for it seemed to imply a handedness of space which would then not be the empty thing which (since the demise of the ether hypothesis) most physicists think it to be. That is the ether would be needed to provide a standard of handedness at each point of space, to tell Co^{60}, etc., which direction to decay into. But this is not really the situation; the saving thing is that anti-Co^{60} decays in the opposite direction. Thus, after all, there is nothing intrinsically left-handed about the world, just as there is nothing intrinsically positively charged about the nuclei. What really exists here is a correlation between handedness and sign of charge.'

With Dr. Goebel's position we are in complete agreement. (I use the pronoun 'we' when I feel I can probably include the reader, and the pronoun 'I' when I am expressing a more controversial personal opinion.) The handedness of the ψ-wave constituting a particle and the sign of charge of that particle are one and the same thing.

It is true of course that we live in a part of the universe where electrons are much more plentiful than positrons, or matter than anti-matter, but that is the result of the fact that matter and anti-matter must separate to survive, and only the matter and anti-matter that have separated still survive. We have to live in a region of

matter or in a region of anti-matter, not in a mixture of both.

If we are to adhere to our belief that there is nothing intrinsically or preferentially right- or left-handed about space, a belief which is supported by the field equation, then we are obliged to postulate that equal amounts of matter and of anti-matter exist in the universe, and it is fortuitous that we happen to live in a region of matter rather than in a region of anti-matter. It remains to be seen how large these regions are, and whether on the average alternate stars, or galaxies, or larger regions, are composed alternately of matter and of anti-matter.

Neutral particles might exist either because they are symmetrical and have no handedness, or because even though they are unsymmetrical the total amount of left-handed field energy equals the total amount of right-handed field energy, as in a proton-electron pair. A neutron appears to be of the latter type, since a distinguishable anti-neutron exists. If the neutron were symmetrical it would be identical with the anti-neutron and indistinguishable from it. It seems that symmetrical particles or symmetrical combinations of particles, like electron and positron, or proton and anti-proton, are unstable and disintegrate into electromagnetic radiation. It takes an unsymmetrical pair like electron and proton, or positron and anti-proton to be stable, I suspect because they cannot disintegrate without violating the conservation of angular momentum.

When what appears to be a fatal defect in a theory turns out to be a good explanation of hitherto inexplicable facts, one's confidence in the theory is enhanced. When the apparent defect of the field equation, of equating electric charge, which was thought to be positive or negative, to field energy, which is only positive, turned out to give a simple explanation of why charge appears to have two signs, and why parity and sign of charge are related, and why matter and anti-matter exist, and why particles of matter and anti-matter have opposite signs of charge, and why empty space is not preferentially handed but specific ψ-waves constituting matter and anti-matter particles have specific handedness, my confidence started to grow that the field theory is correct.

THE field equation is a succinct statement of the fundamental postulate of quantum mechanics. Since $\frac{1}{2}(H^2+E^2)$ is scalar energy, $E\times H$ is vector momentum, and $\frac{1}{2}\psi^*\iota\psi=-\iota\frac{1}{2}(H^2+E^2)+E\times H$, the field equation $D\psi=\frac{1}{2}\psi^*\iota\psi$ can be written in the form:

$$(\iota\frac{\partial}{\partial t}+\nabla)\psi=\text{scalar energy}+\text{vector momentum}$$

In words, this is the fundamental postulate of quantum mechanics: the scalar energy and the vector momentum of a differential element of volume can be replaced by the differential operators $\iota\partial/\partial t$ and ∇, operating upon the wave function ψ.

Conversely, if one starts with the fundamental postulate and interprets it properly, one obtains the field equation and the unified field theory.

By interpreting it properly I mean interpreting the operation of $\iota\partial/\partial t+\nabla$ on ψ as a quaternionic multiplication with ψ a three-dimensional complex vector. With such an interpretation one finds that the equation $(\iota\partial/\partial t+\nabla)\psi=0$ is Maxwell's equation for free space. This forces one to identify the probability amplitude vector ψ with the complex electromagnetic vector $H+\iota E$. With this identification made, the scalar energy and vector momentum in free space, derived from Maxwell's equations for free space, are $\frac{1}{2}(H^2+E^2)$ and $E\times H$, which, written in terms of $\psi=H+\iota E$ are $\frac{1}{2}\psi^*\iota\psi$. Finally, insertion of $\frac{1}{2}\psi^*\iota\psi$ in place of the words 'scalar energy and vector momentum' in the fundamental postulate of quantum mechanics gives the field equation $D\psi=\frac{1}{2}\psi^*\iota\psi$, and the derivation is complete.

It may be noted that the derivation does not depend upon matter or the existence of matter. It depends only on the fundamental postulate of quantum mechanics and Maxwell's equations for empty space. The field and the field equation depend only on the properties of a complete vacuum. They create matter out of the properties of a vacuum. They are not created by the properties of matter. This is the final dematerialization of physics:

Existence depends only on the properties of a complete vacuum.

DIRAC'S equation is a linear, relativistically invariant, quantum mechanical equation for the probability amplitude ψ of an electron, and of a positron, the existence of which it predicted. In quaternionic notation and in the natural system of units, Dirac's equation is

$$D\psi = K\psi$$

where K is a constant independent of ψ. Dirac's theory gives the charge and current density to be

$$-\iota\rho + \rho v = \tfrac{1}{2}\psi^*\iota\psi$$

What Dirac's theory does not do is to reconcile the two equations above with Maxwell's equation which is

$$D\psi = -\iota\rho + \rho v$$

In order to reconcile Dirac's and Maxwell's equations we postulate that they are both linear approximations to an exact non-linear equation obtained by inserting the expression $\tfrac{1}{2}\psi^*\iota\psi$ for charge and current density in place of $-\iota\rho + \rho v$ in Maxwell's equations or in place of $K\psi$ in Dirac's equation to obtain the field equation

$$D\psi = \tfrac{1}{2}\psi^*\iota\psi$$

In so doing we have not sacrificed Dirac's primary objective, which was to obtain a relativistically invariant equation, which Schroedinger's is not. The field equation is relativistically invariant, as are its two linear approximations, Dirac's equation and Maxwell's equation.

We have however sacrificed Dirac's secondary objective, which was to obtain a linear differential equation to represent a non-linear relativistic Hamiltonian. Dirac's equation is linear, while the field equation is non-linear. In my opinion, it is impossible to represent a non-linear universe exactly by a linear equation since all the non-linear properties are necessarily lost, so Dirac's equation cannot be better than an approximation. If exactness is desired, Dirac's secondary objective is hopeless.

A derivation of Dirac's equation is given in Morse and Feshbach's *Methods of Theoretical Physics* on page 257. The basic idea is to form the scalar product of the four-vector $p_x + p_y + p_z + p_t/c$ with

the Pauli spin four-vector $\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4$, since the scalar product of two four-vectors is Lorentz invariant. The p's are to be replaced by differential operators according to the fundamental postulate of quantum mechanics, and the resulting operator is to be inserted in a linear eigenvalue equation of the form suggested by Schroedinger's equation

$$(\text{Operator})\psi = (\text{Constant})\psi$$

Doing so, one obtains, as on page 258

$$\frac{\hbar}{\iota}\left(\sigma_1\frac{\partial}{\partial x} + \sigma_2\frac{\partial}{\partial y} + \sigma_3\frac{\partial}{\partial z} + \sigma_4\frac{\partial}{c\partial t}\right)\psi = (\text{Constant})\psi$$

Now, as pointed out by Morse and Feshbach on page 104, the Pauli spin operators are related to the Hamiltonian quaternion operators by the relations

$$\iota\sigma_1 = i,\ \iota\sigma_2 = j,\ \iota\sigma_3 = -k' = +k,\ \sigma_4 = 1$$

($-k'$ or $+k$ depends on one's choice of right-handed or left-handed axes, which is arbitrary). Therefore the Pauli four-vector $\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4$ is identical with the complex quaternion $-\iota i - \iota j - \iota k + 1$. If we replace the Pauli spin matrices in the equation by their equivalent unit quaternions, divide by $-\hbar$, and use units in which $c = 1$, the equation becomes

$$\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} + \iota\frac{\partial}{\partial t}\right)\psi = (\text{Constant})\psi$$

or

$$D\psi = K\psi$$

At this point, if one is aware that the equation $D\psi = 0$, $D\psi = C$, or $(i\partial/\partial x + j\partial/\partial y + k\partial/\partial z + \iota\partial/\partial t)\psi = -\iota\rho + \rho v$ is Maxwell's equation, where $\psi = H + \iota E$, one is practically forced to identify the probability amplitude vector ψ with the complex electromagnetic vector $H + \iota E$. One might not be aware if one had not happened to read the great but little known work of L. Silberstein, his *Theory of Relativity*, Macmillan, London, 1914, where he develops relativity and electromagnetic theory in terms of complex quaternions, and shows their superiority over Minkowski's matrices and Sommerfeld's four- and six-vectors, especially in bringing out in a natural way the asymmetries of space-time and of Maxwell's equations.

It is doubtful however whether Dirac would have rested content to use the equation $D\psi = K\psi$ with the Pauli or quaternionic operators since both σ_4 and ι are commutative, and it was Dirac's second-

ary purpose to obtain an equation in which all the operators are non-commutative in order to get a linear equation for a non-linear Hamiltonian, a purpose we have abandoned.

In any case ψ was not set equal to $H+\iota E$, but was set equal to $e+f$, where e is a vector in a spin space of two complex dimensions and f is a vector in a second spin space of two complex dimensions perpendicular to the first. On this assumption, new base matrices or operators, $\alpha_x, \alpha_y, \alpha_z$, and α_o are derived, which are different from either the Pauli or the quaternionic operators. In terms of these the equation $D\psi=K\psi$ becomes Dirac's equation as shown on page 264 of Morse and Feshbach:

$$\alpha_o mc\psi+\frac{\hbar}{\iota}\alpha\cdot\text{grad}\psi=-\frac{\hbar}{\iota c}\frac{\partial\psi}{\partial t}$$

where

$$\psi=\psi_1 e_1+\psi_2 e_2+\psi_3 e_3+\psi_4 e_4$$
$$\alpha_x^2=\alpha_y^2=\alpha_z^2=\alpha_o^2=1$$
$$\alpha_x\alpha_y+\alpha_y\alpha_x=\alpha_y\alpha_z+\alpha_z\alpha_y=\alpha_z\alpha_x+\alpha_x\alpha_z=0$$
$$\alpha_x\alpha_o+\alpha_o\alpha_x=\alpha_y\alpha_o+\alpha_o\alpha_y=\alpha_z\alpha_o+\alpha_o\alpha_z=0$$

and the e's are orthogonal unit vectors in a spin space of four complex or eight real dimensions.

By comparison the quaternionic equation $D\psi=K\psi$ or $(\iota\partial/\partial t+\nabla)\psi=K\psi$ becomes, on multiplying by $-\hbar$ and reintroducing c due to c.g.s. units

$$\hbar K\psi-\hbar\nabla\psi=-\frac{\hbar}{\iota c}\frac{\partial\psi}{\partial t}$$

The identity is clear if it is permissible to let $K=\alpha_o mc/\hbar$, $\nabla=\iota\alpha\cdot\text{grad}$, and $\psi=H+\iota E$. We can let K be whatever constant we please, ∇ used quaternionically must equal $\iota\alpha\cdot$grad since the latter is derived from the equation $D\psi=K\psi$, and the only assumption being made is that ψ is a complex three-vector instead of a four-vector, but would of course appear as a four-vector if expressed in terms of arbitrary base vectors such as the e's, just as a plane in three-space only reduces to two dimensions in a special set of coordinates located in the plane but in arbitrary base vectors appears to be a three-dimensional figure.

By way of further comparison, whereas the α's are square roots of plus one, the quaternionic operators are square roots of minus one.

$$\alpha_x^2=\alpha_y^2=\alpha_z^2=\alpha_o^2=1$$
$$i^2=j^2=k^2=\iota^2=-1$$

Both $\alpha_x, \alpha_y, \alpha_z$ and i,j,k are non-commutative:

$$\alpha_x\alpha_y+\alpha_y\alpha_x=\alpha_y\alpha_z+\alpha_z\alpha_y=\alpha_z\alpha_x+\alpha_x\alpha_z=0$$
$$ij+ji=jk+kj=ki+ik=0$$

But α_o is non-commutative while ι (or Pauli's σ_4) is commutative:

$$\alpha_x\alpha_o+\alpha_o\alpha_x=\alpha_y\alpha_o+\alpha_o\alpha_y=\alpha_z\alpha_o+\alpha_o\alpha_z=0$$
$$i\iota+\iota i\neq j\iota+\iota j\neq k\iota+\iota k\neq 0$$

It is the non-commutative property of α_o that allows Dirac to linearize the non-linear Hamiltonian since it is this property that causes all the non-linear cross-products to disappear. The quaternionic and Pauli operators won't do this, but we do not mourn the loss since we believe that the basic linear equation (Operator)ψ = (Constant)ψ postulated by Dirac is only an approximation anyway. Linearity and exactness are incompatible.

The α's may be represented as four-by-four matrices with complex elements (or eight-by-eight matrices with real elements):

$$\alpha_o=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\qquad \alpha_x=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix}$$

$$\alpha_y=\begin{pmatrix}0&0&0&-\iota\\0&0&\iota&0\\0&-\iota&0&0\\\iota&0&0&0\end{pmatrix}\qquad \alpha_z=\begin{pmatrix}0&0&1&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{pmatrix}$$

Similarly the quaternionic units can be represented as four-by-four matrices with complex elements:

$$1=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\qquad i=\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}$$

$$j=\begin{pmatrix}0&0&-1&0\\0&0&0&1\\1&0&0&0\\0&-1&0&0\end{pmatrix}\qquad k=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}$$

The imaginary quaternionic units ι, ιi, ιj, and ιk are the above multiplied by ι, or with each 1 replaced by ι.

The advantage and purpose of the α representation is that the equation becomes

$$(\alpha_x p_x+\alpha_y p_y+\alpha_z p_z+\alpha_o mc)\psi=\frac{p_t}{c}\psi$$

where the p's are differential operators, and the equation can be squared, whereupon due to the non-commutation of all the α's including α_o, the cross-products drop out and the result can be identified, by the fundamental postulate of quantum mechanics, with the non-linear relativistic Hamiltonian, where the p's are classical generalized coordinates

$$p_x^2+p_y^2+p_z^2+m^2c^2=p_t^2/c^2$$

Thus a linear equation represents a non-linear Hamiltonian. In the quaternionic or Pauli representation not all the operators are non-commutative, so the cross-products do not cancel out.

The disadvantage of the α representation is that the cyclic relations ij=k, etc., are lost and with them the simple quaternionic algebra that enables one to identify $D\psi=0$ with $-\nabla\cdot H-\iota\nabla\cdot E+(\nabla\times H-\partial E/\partial t)+\iota(\nabla\times E+\partial H/\partial t)=0$ or Maxwell's equation for free space, and so identify ψ as both the probability amplitude vector and the complex electromagnetic vector. Furthermore spin space, as defined by the α's, is not obviously related to physical space-time, whereas quaternionic space clearly contains space-time as a sub-space along the ι, i, j, and k axes. Yet again, by using the α's one does not get rid of the quaternionic base units 1, ι, i, j, and k, which all appear in Dirac's equation, as in ιgrad, (although they are not generally thought of as quaternions), so that Dirac's equation contains a rather complicated mixed base system of the α's, the e's, and the quaternionic units, instead of just the quaternionic units alone. It is the hopeless quest for linearity that leads to the obscure and complex α representation instead of to the clear and simple quaternionic representation.

The really serious objection to Dirac's equation, however, is that it does not reconcile wave mechanics with electromagnetism. Once we recognize that $D\psi=-\iota\rho+\rho v$ is Maxwell's equation, and $D\psi=K\psi$ is Dirac's equation, we cannot but feel that the equations are related and that the more accurate quantum mechanical equation, which is Dirac's $D\psi=K\psi$, should, by the Correspondence Principle, break down into the classical equation, which is Maxwell's $D\psi=-\iota\rho+\rho v$. But it doesn't. It would if Dirac's theory defined charge density ρ and current density ρv or J as being equal to $K\psi$; or $-\iota\rho+\rho v=K\psi$.

But this is not the case. Dirac's theory gives ρ and J as the following, given on page 261 of Morse and Feshbach:

$$\rho=e\bar{\psi}\psi=e(|\psi_1|^2+|\psi_2|^2+|\psi_3|^2+|\psi_4|^2)$$
$$J_x=ce(\bar{\psi}\alpha_x\psi)=ce(\bar{\psi}_1\psi_4+\bar{\psi}_2\psi_3+\bar{\psi}_3\psi_2+\bar{\psi}_4\psi_1)$$
$$J_y=ce(\bar{\psi}\alpha_y\psi)=-\iota ce(\bar{\psi}_1\psi_4-\bar{\psi}_2\psi_3+\bar{\psi}_3\psi_2-\bar{\psi}_4\psi_1)$$
$$J_z=ce(\bar{\psi}\alpha_z\psi)=ce(\bar{\psi}_1\psi_3-\bar{\psi}_2\psi_4+\bar{\psi}_3\psi_1-\bar{\psi}_4\psi_2)$$

In quaternionic notation and natural units:

$$-\iota\rho+J=-\iota\rho+\rho v=\tfrac{1}{2}\psi^*\iota\psi$$

Electromagnetism and wave mechanics become reconciled if $K=\frac{1}{2}\psi^*\iota$. But when one lets $K=\frac{1}{2}\psi^*\iota$ in order to put the expression $\frac{1}{2}\psi^*\iota\psi$ for charge and current into Dirac's equation $D\psi=K\psi$ so that, by the Correspondence Principle, it will break down into Maxwell's equation $D\psi=-\iota\rho+\rho v$, one no longer has Dirac's equation but the field equation $D\psi=\frac{1}{2}\psi^*\iota\psi$.

The Relativistic Invariance of the Field Equation

THE theory of special relativity, which has been well substantiated experimentally, demands that the form of the equation representing a physical law remain invariant under a Lorentz transformation from one inertial frame of reference to another moving at a uniform speed with respect to the first. Were this not so there would be a preferred inertial frame of reference in which the law had one form, perhaps an especially simple form, and other frames in which it took on a different and more complicated form. Relativity denies that there is any preferred inertial reference system.

The field equation cannot pretend to be *the* or even *a* fundamental law of physics unless it is relativistically invariant. It is. Furthermore we are not justified in tampering with Dirac's equation, which is relativistically invariant, unless the more general field equation to which we claim Dirac's is an approximation, is also invariant.

L. Silberstein, in his *Theory of Relativity*, Macmillan, London,

1914, develops Einstein's theory in terms of complex quaternions. In view of the availability of Silberstein's thorough treatment of the subject, I do not need to repeat it but will simply summarize it.

Silberstein uses, in place of a four-vector, a complex quaternion of the form $q=\iota a+b$ or $q_c=\iota a-b$, where a is a scalar and b is a three-vector. He calls q or q_c a 'physical quaternion'. The archtype is an interval in space-time $\iota t+r$. Other examples are the charge quaternion $-\iota\rho+\rho v$ and the energy quaternion $\frac{1}{2}\psi^*\iota\psi=-\iota\frac{1}{2}(H^2+E^2)+E\times H$. He also introduces the complex electromagnetic vectors, which he calls R and L, and which we call ψ and ψ^*, as $\psi=R=H+\iota E$ and $\psi^*=L=H-\iota E$. He also defines the quaternionic differential operators which he calls D and D_c but which we call $\bar{D}$ and $\bar{D}_c$ to distinguish from our operators D and D_c which are slightly different. Setting $c=1$, $\bar{D}=-\iota\partial/\partial t+\nabla$ and $\bar{D}_c=-\iota\partial/\partial t-\nabla$, so our $D=\iota\partial/\partial t+\nabla=-\bar{D}_c$ and our $D_c=\iota\partial/\partial t-\nabla=-\bar{D}$. He then develops the Lorentz transformation from reference system S to reference system S′, which is moving at a uniform velocity v with respect to S. If u is a unit vector in the direction of v, $\beta=v/c$, and $\omega=\arctan\iota\beta$, he shows that the Lorentz transformation of physical quaternions, differential operators, and complex electromagnetic vectors can be expressed in terms of the transformation or rotator quaternions

$$Q=\cos\tfrac{1}{2}\omega+u\sin\tfrac{1}{2}\omega$$
$$Q_c=\cos\tfrac{1}{2}\omega-u\sin\tfrac{1}{2}\omega$$
$$QQ_c=Q_cQ=1$$

The transformations from the unprimed system S to the primed system S′ are

$q'=QqQ$		
$q_c'=Q_cq_cQ_c$		
$D'=Q_cDQ_c$ or in Silberstein's notation	$-\bar{D}_c'=Q_c(-\bar{D}_c)Q_c$	
$D_c'=QD_cQ$	"	$-\bar{D}'=Q(-\bar{D})Q$
$\psi'=Q\psi Q_c$	"	$R'=QRQ_c$
$\psi^{*\prime}=Q_c\psi^*Q$	"	$L'=Q_cLQ$

The unit imaginary scalar ι is a special case of $q=\iota a+b$ or of $q_c=\iota a-b$, where $a=1$ and $b=0$. It transforms like a pure time interval, either as $\iota'=Q\iota Q$ or $\iota'=Q_c\iota Q_c$, whichever is convenient. Real scalars remain invariant. Using these transformations and inserting $Q_cQ=QQ_c=1$ wherever we wish we can transform the field

equation from reference system S to reference system S′.

In S:

$$D\psi=\tfrac{1}{2}\psi^*\iota\psi$$

Premultiplying and postmultiplying by Q_c, and inserting $QQ_c=Q_cQ=1$, the field equation becomes

$$Q_cDQ_cQ\psi Q_c=\tfrac{1}{2}Q_c\psi^*QQ_c\iota Q_cQ\psi Q_c$$

Using the transformations for each factor the equation becomes:

In S′:

$$D'\psi'=\tfrac{1}{2}\psi^{*\prime}\iota'\psi'$$

The form of the field equation is the same in S′ as in S, so it is relativistically invariant.

A point to note is that the position of ι in $\frac{1}{2}\psi^*\iota\psi$ is important. It is *not* $\frac{1}{2}\iota\psi^*\psi$, although it can be written that way if one stays in the system S. While ι is a commutative scalar in the given reference system S, and ι' is in S′, ι is not a commutative scalar in a Lorentz transformation but a special case of a non-commutative quaternion $q=\iota a+b$ or $q_c=\iota a-b$, where b in the given reference system happens to be zero, but is not zero to a moving observer. It's just like any pure time interval under a Lorentz transformation, which ceases to be a pure commutative time scalar and picks up a non-commutative spatial vector component. For this reason the expanded form of $\frac{1}{2}\psi^*\iota\psi$, namely $-\iota\frac{1}{2}(H^2+E^2)+E\times H$, to get which ι is treated as commutative, is not relativistically invariant, although $\frac{1}{2}\psi^*\iota\psi$ is. $\frac{1}{2}\psi^*\iota\psi$ is invariant in essentially the same way that Dirac's expressions for charge and current, such as $J_x=ce\bar{\psi}\alpha_x\psi$, are invariant and where again the order of the factors such as α_x is important.

The relativistic invariance of the field equation is an extremely convincing piece of evidence that the unified field theory is correct. For, with the possible exception of one or two equations like the Klein-Gordon equation which have not been too successful in explaining physical phenomena, the *only* relativistically invariant equations are the field equation $D\psi=\frac{1}{2}\psi^*\iota\psi$, Dirac's equation $D\psi=K\psi$, and Maxwell's equation $D\psi=C$; and the last two are approximations of the first. The burden therefore rests upon the skeptic to produce an alternative invariant equation that accounts for Dirac's and Maxwell's equations, or disprove relativity; or accept the field equation.

A Qualitative Explanation of Relativity

IT is a tribute to the extraordinary genius of Albert Einstein that he was able to develop the theory of special relativity by sheer logic, although all the intuitive pictures of what constituted physical reality, according to the theories of the time, denied relativity. But when an intuitive picture disagrees with the facts, the picture must be wrong. The facts can't be wrong. One of the strongest arguments in favor of the unified field theory is that it presents an intuitive picture of physical reality that is compatible with relativity and makes at least some of the conclusions of the theory of relativity seem obviously true.

The limiting speed of a particle is the speed of light because a particle *is* light.

What explanation could be simpler? If a particle were a corpuscular bullet why should there be any connection between its speed and the speed of a wave predicted by Maxwell's equations? But if the particle is itself a wave predicted by a simple generalization of Maxwell's equations, namely the field equation, what else would one expect?

It requires an infinitely large addition of energy to a particle to accelerate it to the speed of light. A particle having rest mass is a standing wave packet, composed, along any arbitrary axis, of a forward wave and a backward wave. To make the packet move, energy must be added to the forward wave, which slowly shifts the position of the maximum wave of the standing wave packet: that is, the particle slowly moves. The more energy added the faster the particle goes. But energy is not subtracted from the backward wave component which remains at its original magnitude and slows down the forward wave. It takes a free wave, i.e. a pure forward or backward wave, to travel with the speed of light. Therefore one would have to add an infinite amount of energy to the forward wave component of a particle to make the backward component relatively negligible, so that the particle consisted effectively of a pure forward wave, or free wave, travelling at the speed of light.

Rest mass is the square of the amplitude of the standing ψ-wave

constituting a rest-mass particle. Energy is the square of the amplitude of a free travelling ψ-wave, as in classical electromagnetic theory. A standing wave consists of a forward travelling wave and a backward travelling wave which cancel each other's motion. Nonlinear interaction cements the two travelling waves together, so that the incoming wave builds up its energy at the expense of the outgoing wave until it reaches the center of the particle, after which the direction of energy flow is reversed and what was the increasing incoming wave becomes the decreasing outgoing wave. If the nonlinear interaction which locks the two travelling waves together becomes unstable and is overcome, the standing wave separates into its two constituent travelling waves. Mass becomes energy. A positronium particle, which is the combination of an electron and a positron, is unstable and disintegrates into two travelling electromagnetic waves. The rest mass of the positronium particle, which is the square of the amplitude of the standing wave, becomes the energy or the squares of the amplitudes of the two travelling waves. The equivalence of mass and energy is obvious.

The theory of relativity proclaims that each of two observers, in relative motion with respect to each other at a uniform velocity, finds that he is at rest with regard to the ψ-field or with regard to the vacuum, and to each light appears to travel at the same constant speed c. The explanation is that from the viewpoint of the first observer the yardsticks of the second have shrunk and his clocks have slowed down by virtue of his velocity so that the second cannot detect his own velocity with respect to the field. The situation is reciprocal and the second observer comes to the same conclusions about the first. The changes in yardsticks and clocks are described by the Lorentz transformations. The changes are inexplicable if the yardsticks and clocks are composed of point particles or billiard balls, but are explained by the field picture of wave packet particles. A yardstick is the wavelength of a standing wave or its multiple. A clock period is the period of a standing wave or its multiple. From the viewpoint of the first observer, whom we will call stationary, the standing wave yardsticks and clocks of the second observer are moving, or have had their forward wave components augmented in relation to their backward wave components in order to move. The increment in the forward wave in a standing wave packet changes

the wavelength and period of the packet, by the amount indicated by the Lorentz transformations. The unified field theory presents a process of wave addition as the explanation and mechanism of the Lorentz transformation.

The theory of relativity applies universally to all physical entities: particles, macroscopic bodies, and electromagnetic waves. Unless they are all essentially the same entity why should this be? One can see that it applies to electromagnetic waves because Maxwell's equations are relativistically invariant, but why should it apply to corpuscular point particles which are not solutions of Maxwell's equations or anything like Maxwell's equations? But if particles, macroscopic bodies, and electromagnetic waves are all standing or travelling waves in the ψ-field and are all solutions of the field equation $D\psi = \frac{1}{2}\psi^* \iota\psi$ which is relativistically invariant, then the universality of relativity is obvious.

The Fundamental Non-Existence of Mechanical Motion

THE central theme of Newtonian physics was mechanical motion. Classically it was thought that matter and space (or particles and fields) were two fundamental entities. A particle was a closed shape like a sphere which contained matter inside its surface and was surrounded by empty space outside its surface. The ultimate nature of matter, which filled the sphere like ivory, was beyond conjecture: it simply existed as a fundamental entity. Mechanical motion was thought of as the motion of the sphere with respect to other spheres, and, until relativity, with respect to empty space.

According to the unified field theory mechanical motion, at the fundamental microscopic level, does not exist. The field vector ψ is the only physical reality, and it does not move. ψ is a magnitude, or rather a set of six magnitudes H_x, H_y, H_z, ιE_x, ιE_y, and ιE_z at each point of space and time. ψ changes from point to point in space and time, but it is meaningless to say that the ψ-vector at one point

'moves' to another point. If at one moment there is a large ψ at point A and a small ψ at point B, and a moment later the reverse is true, one does not say that the ψ-vector at A has moved mechanically to B, and vice versa. One says that each ψ-vector has remained stationary but has changed its magnitude as the result of a ψ-wave that has moved from A to B or from B to A.

Existence consists of waves in a stationary field. Particles are propagated as waves, like classical light waves. They are not transported as corpuscles, like miniature cannon balls. The relativistic invariance of the field equation ensures that the field remains stationary to all inertial observers.

The Derivation of Newton's Laws of Motion and Classical Mechanics

IT is well known in the classical theory of electromagnetism that $(1/c)E\times H$ is momentum and $(1/c)\partial(E\times H)/\partial t$ is force. Electromagnetic radiation has momentum and exerts forces. The unified field theory claims that, in units where $c=1$, *all* momentum is $E\times H$, and *all* inertial forces, corresponding to Newton's $d(mv)/dt$, are $\partial(E\times H)/\partial t$.

The historical development went as follows. First force F was defined. Then Newton's law of motion $F=d(mv)/dt$, Maxwell's equations, and the electromagnetic force law $F=\rho(E+(1/c)v\times H)$ were postulated. Then Maxwell's equations were used to eliminate ρ and ρv from the electromagnetic force law. One arrives at an expression for an integrated force K, integrated over a volume τ, where

$$K=\int_\tau F\,d\tau$$

and finds

$$K=-\frac{1}{c}\frac{\partial}{\partial t}\int_\tau (E\times H)\,d\tau$$

To be consistent with Newton's law, $(1/c)E\times H$ must be interpreted as momentum. Details are given in L. Page's *Introduction to Theoretical Physics*, pages 441–447.

It seems to me that this argument can be used backwards so that, if we initially postulate the unified field theory, we derive Newton's law from it.

First we postulate the field equation and its constant approximation, Maxwell's equations, and we define $E\times H$ as momentum. By purely mathematical manipulations, as shown in Page, page 442, we obtain an expression equal to $\rho(E+(1/c)v\times H)$. We define this quantity to be what we mean by force F. We then integrate over a volume to get the total force K on all the classical particles in τ exerted by the rest of the field in τ. The integrated expression contains a number of surface integrals that vanish at the boundary of τ if the field vanishes there, and we are left with

$$K=-\frac{1}{c}\frac{\partial}{\partial t}\int_\tau (E\times H)\,d\tau$$

This is Newton's law of motion. It states that the force K on the classical particles or small control spheres in τ is equal and opposite to the force on the rest of the field in τ excluding the classical particles, and that the force is equal to the time rate of change of momentum of the field.

Newton's law can be extended to the case of forces of action and reaction exerted by two material bodies, or collections of classical particles, on each other, by simply including the particles composing the second body in what we have called above 'the rest of the field in τ'. This is legitimate since, according to the field theory, particles are just intense regions of the field and classical particles are fictitious spherical control surfaces placed around such regions.

It may be noted that when ψ, equal to $H+\iota E$, changes sign, $E\times H$ does not change sign but $D\psi$ does. Hence inertial forces, involving $\partial(E\times H)/\partial t$, are essentially positive, while electromagnetic forces, involving $D\psi$, are positive or negative.

Given Newton's law of motion, the rest of classical mechanics follows at once by purely mathematical deduction, including Lagrange's equations, Hamilton's equations, Hamilton's principle, and so forth.

THE conservation laws are implicit in the field equation. If one multiplies $\nabla\times E+\partial H/\partial t=0$ by H, and $\nabla\times H-\partial E/\partial t=E\times H$ by E, and subtracts the second from the first, one obtains:

$$\frac{\partial}{\partial t}\left\{\tfrac{1}{2}(H^2+E^2)\right\}+H\cdot\nabla\times E-E\cdot\nabla\times H+E\cdot E\times H=0$$

Now $$H\cdot\nabla\times E-E\cdot\nabla\times H=\nabla\cdot(E\times H)$$

and $$E\cdot E\times H=0$$

Hence $$\frac{\partial}{\partial t}\left\{\tfrac{1}{2}(H^2+E^2)\right\}+\nabla\cdot E\times H=0$$

This is the general conservation law. If $E\times H$ is divergenceless, i.e. has no sources or sinks, then $\frac{1}{2}(H^2+E^2)$ does not vary with the time, or is conserved, and vice versa. Or if one integrates over a volume τ having a surface σ and applies Gauss' theorem:

$$\frac{\partial}{\partial t}\int_\tau \tfrac{1}{2}(H^2+E^2)\,d\tau+\int_\sigma E\times H\cdot d\sigma=0$$

The only way a volume can change its store of $\frac{1}{2}(H^2+E^2)$, which is its scalar energy-mass-charge-probability, is to transmit or receive $E\times H$ across its boundary, where $E\times H$ is vector energy flux-momentum-electric current-probability current. The separate conservation laws applying specifically to energy, mass, charge, momentum, etc., are special cases or interpretations of this general law.

The general conservation law, derived from the field equation, differs from Poynting's theorem, derived from Maxwell's equations, in one important respect. The term $E\cdot E\times H$ in the field theory is zero, while the corresponding term $E\cdot J$ in Poynting's theorem is the non-zero Joule heating loss. The ψ-field, at the fundamental level before it is organized into particles and macroscopic bodies, is lossless. There is no Joule heating: there is nothing to heat. Physicists have always assumed that elemental processes are lossless, whether they were picturing elastic collisions of classical particles, planetary atoms, or oscillatory wave-mechanical atoms. The assumption was a postulate however: it had no theoretical justification. Now it has.

THE field equation, with its initial and boundary conditions, is self-sufficient, and therefore cannot tolerate any independent equations or laws. An independent equation has no way of affecting a universe described and determined by the field equation. Gravitational and nuclear fields therefore cannot be independent fields governed by separate equations. They must be ψ-fields governed by the field equation.

The qualitative picture that the field theory presents of gravitational and nuclear fields is that they are ψ-fields under special conditions.

Gravitational fields are ψ-fields, i.e. electromagnetic fields, between neutral particles.

Nuclear fields are ψ-fields over distances which are comparable to the wavelengths of the standing ψ-waves that constitute the nucleus, and are sufficiently intense so that the non-linearity of the field becomes dominantly important.

We are used to thinking of the radio field and the induction field as two cases or terms of the electromagnetic field, not as two totally different kinds of fields governed by different laws. We now extend this idea to gravitational and nuclear fields, which are also special cases or terms of the electromagnetic field.

Such an extension was not possible as long as we thought of energy-mass as one fundamental entity and electric charge as another fundamental entity. The gravitational field had its sources in energy-mass, while the electromagnetic field had its sources in electric charge, and there was no relation between the two. But the field theory identifies energy-mass with charge as being the single entity energy-mass-charge $\frac{1}{2}(H^2+E^2)$. With this identification made the gravitational and electromagnetic fields have a common source and become a single entity.

We can still distinguish the fields due to charged particles from those due to neutral particles. The fields which we are accustomed to think of as electromagnetic are those that exist between charged bodies. There are also fields, which we still think of as electro-

magnetic, between a charged body and a neutral one, such as those that produce a force on a dipole in an inhomogeneous field. These forces are down an order of magnitude from the forces between two charged bodies. Down still further are the fields and forces between two neutral bodies, such as a dipole in the field of another dipole. These are the gravitational fields and forces.

Similarly nuclear fields must be ψ-fields, but under special circumstances. Nuclear and particle fields bear the same relationship to macroscopic electromagnetic fields that physical optics bears to geometrical optics: one is getting down to dimensions comparable to wavelengths. Also, and even more important, one is getting up in intensities to rest mass energy densities. One must take into account not only linear interference effects, as is done in classical physical optics and in wave mechanics, but one must also take into account the non-linear interactions introduced by the non-linear field equation. It is the non-linear interactions that promise to explain the existence, stability, and structural details of the nucleus and the elementary particles.

A quantitative description of gravitational and nuclear fields remains to be worked out.

One may speculate on how a quantitative theory of gravitation might be derived from the field theory. One possible avenue is to consider the integral given previously for Newton's Law of Motion. The expression given was not complete. The complete expression, as given by L. Page, *Introduction to Theoretical Physics*, 3rd Edition, page 534 (the previous page numbers were for the 1st Edition) is:

$$K=-\frac{1}{c}\frac{\partial}{\partial t}\int_\tau E\times H d\tau+\int_\sigma (EE+HH)\cdot d\sigma-\tfrac{1}{2}\int_\sigma (E^2+H^2)\,d\sigma$$

The surface integrals were assumed to be zero on the basis that the field falls off inversely with the square of the distance and so becomes negligible at a sufficient distance.

However I suspect these surface integrals represent the gravitational force. If the earth were surrounded by a surface σ between it and the sun where the field ψ was everywhere zero and remained zero, then according to the field theory in which ψ is the only physical reality, the earth would be a separate universe completely isolated from the sun, and would not and could not know that the sun existed. There would be no gravitational force between the

earth and the sun. It is the non-zero field at σ that connects the earth and the sun, and transmits the gravitational force.

Consider all the positively charged particles constituting the earth to be concentrated at its center. They would produce an E-field and perhaps an H-field which, just from geometry, would fall off inversely with the square of the distance out to the surface σ. At this surface they would produce a force, as given by the surface integrals. A similar force would be produced at σ by the positive particles constituting the sun. But the integrals are proportional to $\frac{1}{2}(E^2+H^2)$, or, in natural units, to the masses of the positive particles composing the earth and the sun. Furthermore, this force, being quadratic in E and H, is essentially positive and does not change sign when, to get negative particles, $\psi=H+\iota E$ is replaced by $-\psi=-H-\iota E$. Hence the negatively charged particles in the earth and the sun would double the force, not cancel it. We thus have an avenue opened up to us to explain a force which is essentially positive, falls off with the inverse square of the distance, and is proportional to the masses of the attracting bodies.

This approach to gravity is attractive from the point of view of relativity, both special and general, since it gives a combined force K of inertia and gravity which decomposes in a given reference frame into an inertial time-dependent term and a gravitational space-dependent term, but which is essentially a single force.

An alternate approach to gravity, or more generally to the field theory as a whole, is to compare it with Einstein's unified field theory. J. A. Wheeler, in his article, with S. Tilson, "Dynamics of Space-Time", in the Dec. '63 issue of *International Science and Technology*, points out that Einstein's theory is completely summarized in the statement:

Intrinsic curvature–extrinsic curvature $= 16\pi$ *(energy density)*

The field theory embodied in $D\psi=\frac{1}{2}\psi^*\iota\psi$, is summarized in the statement:

Quaternionic rate of change of the field = energy density quaternion

Are the two theories saying the same thing in different mathematics or are they fundamentally different theories?

The Proportionality of Electric Charge Density to Electromagnetic Field Energy Density, and the Derivation of the Field Equation Therefrom

An alternate approach to the unified field theory can be made by postulating that electric charge density is proportional to electromagnetic field energy density, and deriving the field equation from the postulate.

It has long been obvious that, at least in absolute value, charge is very roughly proportional to mass or matter. Charge has its sources in particles of matter, not in empty space. Since relativity has shown that mass and energy are equivalent, charge is roughly proportional to energy. The trouble has been that charge can be positive or negative, while mass and energy are essentially positive. Furthermore charge has not seemed to be exactly proportional to mass as the electron and the proton display the same charge, while the neutron displays none. Secure as we now are in our knowledge that there will be a good explanation for the two signs of charge depending on the handedness of the ψ-fields constituting particles, and that neutral or partly neutral particles can exist if the distribution of ψ that constitutes them is partly right- and partly left-handed causing internal neutralization, we can proceed to postulate that charge density is exactly proportional to field energy density.

We will now prove that a natural system of physical units exists, and that in natural units electric charge density is not only proportional to, but is identical with, field energy density; and that with this identity Maxwell's equations become the field equation.

We postulate that electric charge density ρ is proportional to electromagnetic field energy density $\frac{1}{2}(H^2+E^2)$ with a constant of proportionality k_1, or $\rho=k_1\frac{1}{2}(H^2+E^2)$. If scalar charge is proportional to scalar energy, then vector current should be proportional to vector energy flux, or $\rho v/c=k_2E\times H$, where for the moment k_2 may be different from k_1. Substituting in Maxwell's equations, expressed in Heaviside-Lorentz units:

Maxwell's Equations	*Substituted Equations*
$\nabla\cdot H=0$	$\nabla\cdot H=0$
$\nabla\cdot E=\rho$	$\nabla\cdot E=k_1\frac{1}{2}(H^2+E^2)$
$\nabla\times H-\frac{\partial E}{c\partial t}=\frac{\rho v}{c}$	$\nabla\times H-\frac{\partial E}{c\partial t}=k_2E\times H$
$\nabla\times E+\frac{\partial H}{c\partial t}=0$	$\nabla\times E+\frac{\partial H}{c\partial t}=0$

First we wish to show that $k_1=k_2$. Let us assume we have a plane light wave expressed by E_o and H_o, which is a solution of Maxwell's equations with $\rho=0$. In such a case $E_o\times H_o=\frac{1}{2}(H_o^2+E_o^2)$. We wish to put E_o and H_o into the right-hand non-linear terms of the substituted equations to give an approximate linear solution by iteration, and identify the terms with ρ and ρv in Maxwell's equations, which we consider to be the same approximate linear solution. The $\nabla\cdot E$ equations give $\rho=k_1\frac{1}{2}(H_o^2+E_o^2)$. The $\nabla\times H$ equations give $\rho v/c=k_2E_o\times H_o=k_2\frac{1}{2}(H_o^2+E_o^2)$. If we identify v with c in this case of a light wave, then $\rho=k_2\frac{1}{2}(H_o^2+E_o^2)$. Comparing the two expressions for ρ, we see that $k_1=k_2$.

Incidentally we see that a light wave can act as a source of charge or as an electric current. What the unified field theory essentially claims is that *all* sources of charge and *all* electric currents are light waves, including the standing light-wave packets we call particles and macroscopic material bodies.

Now we wish to eliminate k_1 and c from the substituted equations by selecting new units of, say, time and mass, so that the new measure numbers E', H', and t' are related to the old by $E'=k_1E$, $H'=k_1H$, and $t'=ct$. In other words the new unit of field intensity is $1/k_1$ as big as the old; the new unit of time is $1/c$ as big as the old. Substituting:

$$\nabla\cdot\frac{H'}{k_1}=0$$

$$\nabla\cdot\frac{E'}{k_1}=k_1\frac{1}{2}\left(\frac{H'^2}{k_1^2}+\frac{E'^2}{k_1^2}\right)$$

$$\nabla\times\frac{H'}{k_1}-\frac{\partial E'}{k_1c\partial\frac{t'}{c}}=k_1\frac{E'}{k_1}\times\frac{H'}{k_1}$$

$$\nabla\times\frac{E'}{k_1}+\frac{\partial H'}{k_1c\partial\frac{t'}{c}}=0$$

The constants k_1 and c cancel out, and, dropping the primes, we have:

$$\nabla\cdot H=0$$

$$\nabla\cdot E=\frac{1}{2}(H^2+E^2)$$

$$\nabla\times H-\frac{\partial E}{\partial t}=E\times H$$

$$\nabla\times E+\frac{\partial H}{\partial t}=0$$

These equations constitute the field equation and can be expressed in complex quaternions as

$$D\psi=\frac{1}{2}\psi^*\iota\psi$$

If it is true that electric charge density is proportional to electromagnetic field energy density there is no retreat from a unified field theory, for upon substituting $k_1\frac{1}{2}(H^2+E^2)$ for ρ and $k_2E\times H$ for $\rho v/c$ in Maxwell's equations, the equations close like a sprung trap and become self-sufficient. This means that the equations, if correct, must explain all of physics and all of existence, for they will tolerate no rival laws or equations. For example the laws of mechanics, such as Newton's laws of motion, cannot exist except as they are implied by the field equation, for as independent laws they would have no way of affecting E and H in, say, an electric motor. We know of course that E and H in a motor depend on the torque, inertia, and acceleration of its rotor. All these dependencies must now be implicit in the self-sufficient field equation, which determines how E and H will evolve without any explicit reference to torques, inertias, accelerations, or mechanical laws of motion. I found this exclusive self-sufficiency of the field equation very startling at first, but of course it is essential to any unified theory.

The Natural System of Units

THE field equation, devoid of any physical constants or factors of proportionality like c and k_1, defines a natural system of units, or more precisely a single natural unit. It is to be hoped that the

natural system of units will replace the metric c.g.s., Giorgi M.K.S., and English systems, both for theoretical and practical purposes. For practical purposes it will probably be desirable to use terms like 'micro-units' and 'giga-units' to get numbers of a convenient size.

If the field equation is written in Heaviside-Lorentz units, that is in rationalized Gaussian c.g.s. units mixing e.m.u. and e.s.u. quantities, it contains two factors of proportionality: c, the speed of light in centimeters per second, and k_1, the amount of electric charge equivalent to a unit of energy in rationalized statcoulombs per erg. The natural system of units is obtained by changing the size of two of the three fundamental units of measurement, namely length, time, and mass, to make both c and k_1 equal to unity.

The field equation appears to leave one of the three fundamental units still arbitrary, but such is not the case. The equation is non-linear and therefore must give solutions which have definite numerical values. Each solution represents a physical entity or situation which can be described experimentally in terms of numbers in the c.g.s. system. The ratio of a number obtained theoretically from the field equation to a number obtained experimentally in the c.g.s. system will fix the size of the third fundamental unit of measurement.

For example, suppose we make $c=1$ by picking a new unit of time equal to $1/c$ seconds, and make $k_1=1$ by picking a new unit of mass. This gives us the field equation with no physical constants such as c and k_1 in it. We have not picked a new unit of length, however, so the field equation appears still to be expressed in terms of centimeters, but it is not. Let us suppose we can find the solution of the field equation that describes the hydrogen atom and its spectral lines. The solution will give the wavelength of an identifiable spectral line as a certain number, say n. The wavelength experimentally is known to be m centimeters. We conclude that measure numbers of lengths in natural units are n/m times the measure numbers in centimeters, or the natural unit of length is m/n centimeters long. The third fundamental unit of measurement is thus fixed so the natural system of units is completely determined and definite.

At the moment, the number n/m is not known and the number k_1 is not completely certain. Knowledge and certainty must await our obtaining solutions of the field equation which can be identified with laboratory experiments.

The number k_1 is probably but not certainly the ratio of charge to mass, or charge to rest mass energy (mass and rest mass energy are identical if $c=1$ from $\varepsilon=mc^2$) of an electron, or e/m, expressed in rationalized statcoulombs per erg. The reason we are not certain of this is that it presumes that an electron, unlike a proton or neutron, has no internal neutralization of charge due to part of the ψ-field distribution being right-handed and part left-handed. For example if we tried to find k_1 by using the ratio of charge to mass of a proton the value would be too small. For a neutron it would be zero. If there is any internal neutralization of charge in an electron its ratio will also give a value of k_1 that is too small. For the moment we assume that there is no internal neutralization so $k_1=e/m$. To be sure, we need the solution of the field equation that represents an electron, which we don't have yet.

We could completely determine the natural system of units if, instead of determining the ratio n/m discussed above, we could guess one more constant, like c and e/m, which becomes unity in the natural system. My guess is that Planck's constant h or $h/2\pi$ becomes unity, but this is purely a guess and may be inconsistent with the famous magic number ch/e^2 being 137. It may be possible to use the magic number itself to fix the third fundamental unit but one cannot be sure that the ratio $ch/e^2=137$ should not contain one or more non-dimensional constants like $4\epsilon ch(\sqrt{4\pi}/e)^2=137$, where $\epsilon=2.718$, the Naperian base, and e has been rationalized, in which case $ch/e^2=1$. It is better to get solutions of the field equation.

The field equation, being non-linear, accounts for the fact that physical phenomena, particles, and constants are of fixed and specific sizes, cannot be scaled up or down, and do not depend on the initial conditions of the differential equation. The experimental fact is that the world is that way, and no linear theory can explain it. For that reason, if for no other, it seems to me axiomatic that any fundamental equation of physics must be non-linear.

The field equation $D\psi=\frac{1}{2}\psi^*\iota\psi$ is a purely mathematical equation, devoid of physical constants, units, or dimensions. In principle all physical laws and constants can be found from it by purely mathematical deduction.

The Revision and Possible Experimental Verification of Gauss' or Coulomb's Law of Electrostatics and of Ampere's Law

ACCORDING to the unified field theory, Gauss' Law of Electrostatics $\nabla \cdot E = \rho$, or the equivalent Coulomb's Law of Electrostatics $F = q_1 q_2 / r^2$, and Ampere's Law, as modified by Maxwell, $\nabla \times H - (1/c)\partial E/\partial t = \rho v/c$, are not correct. The correct laws, in Heaviside-Lorentz units, are $\nabla \cdot E = \frac{1}{2} k_1 (H^2 + E^2)$ and $\nabla \times H - (1/c)\partial E/\partial t = k_1 E \times H$, where k_1 is the ratio of charge to rest mass energy of an electron in rationalized statcoulombs per erg.

Gauss' and Ampere's Laws are classical approximations, reasonably good outside classical particles, that is, outside fictitious control surfaces placed around the intense fields that constitute rest masses. They are grossly inaccurate inside classical particles, and even outside they are still only approximations. The revised laws state that electromagnetic fields, even in the absence of charged particles, can act as sources of charge ρ or as electric currents ρv. This suggests that it should be possible, although perhaps delicate, to test the revised laws experimentally to determine if they are correct.

An intense modulated radio or light beam should produce transformer action by the revised Ampere's Law. An intense, coherent light beam, split and mirrored to pass through a region in opposite directions, should produce a zero $E \times H$ but a non-zero $\frac{1}{2}(H^2 + E^2)$ and so according to the revised Gauss' Law should produce an electrostatic field. This is based on the assumption, which may not be valid, that experimental means can be found to isolate solutions of $D\psi = \frac{1}{2}\psi^* \iota \psi$, having one handedness, from solutions of $-D\psi = \frac{1}{2}\psi^* \iota \psi$, having the other handedness, without invoking the use of those existing single-handed solutions, charged particles. Classical light waves, solutions of $D\psi = 0$, do not involve handedness since all terms in the equation change sign when ψ changes sign. Although there may be difficulties, one would hope that, especially with such things as lasers available, the unified field theory can be experimentally verified or refuted.

The Exact Field Equation and Its Solutions

ALTHOUGH we have many approximate solutions of the field equation in the form of solutions of Maxwell's, Schroedinger's, and Dirac's equations, which are approximations to the field equation, we do not have solutions of the exact equation. It is highly desirable that such solutions be obtained. Unfortunately non-linear equations and especially non-linear differential equations are notoriously difficult to solve, and almost no analytical solutions are known. The best method I know by which to attempt to solve the field equation is by numerical methods on a digital computing machine.

I propose to try to solve the eight scalar equations which constitute the field equation by straightforward numerical methods. The differential equations would be replaced by finite difference equations on a finite mesh in four-dimensional space-time. For selected initial conditions and zero boundary conditions the equations would be iterated or allowed to propagate.

The difficulties are at least three.

In the first place, with four independent variables x, y, z, and t, the total number of mesh points varies as the fourth power of the number of points along each axis so that 100 points along each axis would give 100 000 000 mesh points or 800 000 000 equations to solve per iteration. This is beyond the capacity of any present computer. Therefore a rather coarse mesh must be used and initially I propose a very coarse mesh of three points per axis or 81 mesh points or 648 equations per iteration. It remains to be seen whether such a coarse mesh will give significant results.

In the second place, the solutions we particularly seek are the standing waves that constitute the elementary particles. The solutions, or at least certain nodes of the standing waves, must be stationary in space. If the wave moves it will simply propagate away from the mesh points and disappear. On the other hand the solution cannot be constrained to remain stationary by means of boundary conditions or we would be finding the solution of a resonant cavity whereas we want the solution for a free particle. It may be

possible to circumvent this difficulty by selecting certain symmetry conditions so that the wave cannot move in one direction rather than another just from symmetry.

In the third place, there is no guarantee that a solution found is a solution desired, nor that a solution found is a solution identified. If the theory is correct all elementary particles, electromagnetic waves, probability amplitude waves, atoms, molecules, and macroscopic bodies are solutions of the equation. If we want a solution representing one of these we must start with a good approximation to the solution we want. We must know what solution we want and how to recognize it when we get it. We don't know at present. We think however that by plunging in and trying various initial approximations we may obtain and learn to recognize physically significant solutions.

Particles and Their Discrete Properties as Oscillations at Stable Limit Cycles in a Non-Linear Field

WHILE mathematical solutions of the field equation describing the elementary particles are not yet available, the unified field theory presents a qualitative explanation of the elementary particles, of why they have discrete properties, and why one can transmute into another.

The picture the theory presents of a particle having rest mass is that of a standing ψ-wave or wave-packet, of some geometry like one of the spherical harmonics, which is stable, or nearly so, and periodic. Linear wave mechanics, although it attempts to present such a wave-packet picture, is not completely successful because it is not able to account for a wave-packet being stable. A linear wave-packet of finite extent, consisting of a forward wave and a backward wave which, because they are linear do not interact, eventually

comes apart as the forward wave propagates to one side of the universe and the backward wave to the other. It takes a non-linear wave-packet to be stable. Non-linear interaction is the glue of the universe.

A non-linear differential equation typically exhibits the characteristic of having discrete, stable solutions called stable limit cycles. If the system, which might be a non-linear oscillator, is forced to depart from its stable limit cycle by some perturbing forcing function such as an interacting wave it will automatically return upon removal of the forcing function. Characteristically a non-linear system may have a number of stable limit cycles separated by unstable limit cycles. If the perturbing forcing function drives the system to and beyond an unstable limit cycle, then upon removal of the forcing function the system will converge to the *nearest* stable limit cycle, not the one from which it started. The solutions of a non-linear differential equation are typically a set of discrete, separated solutions or limit cycles, rather than a continuum of solutions depending on the initial conditions as in a linear equation.

The field equation is non-linear. It promises to account for the elementary particles as discrete, stable, oscillatory limit cycles in the non-linear field. If the oscillation is forced across an unstable limit cycle to a new stable limit cycle, described by a new geometrical distribution of ψ in space and time, then the particle has been transmuted into a new particle or at least has been excited into a different discrete state. Since there are plenty of possible geometrical distributions of ψ it is not surprising that there are many so-called elementary particles. They are not really elementary. They are various geometrical distributions of the ψ-field, as are all macroscopic bodies, except they are simpler. Only the field vector ψ is elementary.

If a particle or atomic shell is an oscillation at a stable limit cycle in a non-linear field it is easy to see why indistinguishable darticles can exist. Two waves of the same geometry are indistinguishable. It is impossible to put a permanent tag on a wave, although under some conditions an individual wave can be distinguished and its individual history followed for a time.

Similarly it should not be too difficult to account for the Exclusion Principle. The addition of a second electron, i.e. additional

energy, to a given limit cycle would destroy that limit cycle and produce a different one.

Levels of Stability and Unity in the Field: Elementary Particles, Atoms, Molecules, Living Organisms, and Conscious Beings

REST mass particles and macroscopic bodies are more or less stable geometrical distributions of the ψ field vector over some volume of space. They are relatively stable compared to the free ψ-waves like light and radio waves, although probably none is completely stable forever. They fall into a hierarchy of levels of complexity: free waves, elementary particles with rest mass, atoms including nuclei and atomic shells, molecules, living cells, multi-cell organisms, conscious beings, societies of conscious beings, and probably higher levels not yet evolved. At each level a new principle or condition appears which stabilizes and unifies a complex of ψ-waves. The condition we know best is the integral quantum number condition first suggested by DeBroglie and developed by Schroedinger, which unifies an atomic shell. The atomic shells are standing ψ-waves having the geometry of the spherical harmonics, which, to be stable, must have an integral number of wavelengths around a closed circle. If we rule out the possibility of a wave starting at a point on the circle, travelling both clockwise and counterclockwise by two paths around the circle, and leaving the circle at the opposite point, in which case we would be considering travelling waves rather than standing waves; if we rule out a wave which travels around the circle with a different phase at each revolution, in which case we would be considering an unstable wave; and if we rule out the possibility that ψ is a multi-valued function; then there must be an integral number of wavelengths around the circle. This

integral quantum number condition is the foundation of wave mechanics and leads to the description of atomic shells in terms of spherical harmonics, and to the elucidation of the periodic table of chemical elements.

The integral quantum number condition has a stablizing effect and a unifying effect on a portion of the ψ-field. It is a necessary, although insufficient, condition for an atomic shell to be stable. It unifies part of the ψ-field into a new entity, an atomic shell, which has an individuality of its own as a whole not possessed by any single element of volume of the ψ-field composing the shell. The whole is greater than the sum of its parts. Looking at each element of the ψ-field by itself we would not be able to say whether it was part of an atomic shell or not. It is only the properly organized collection of elements of volume, a complete solution of the field equation subject to the integral quantum number condition, that constitutes an atomic shell.

It is supplementary conditions or constraints on the field equation, such as the integral quantum number condition for a stable standing wave, that create discrete and discontinuous atomic configurations, energy levels, and divisions of electric charge from a continuous field. The elemental ψ-field is a continuous and unquantized entity. It seems to me that some people miss the point of DeBroglie's and Schroedinger's wave mechanics when they postulate that the elemental ψ-field itself is somehow quantized, discrete, or corpuscular. The beauty of wave mechanics is that standing waves provide a perfect explanation of how a continuous field can give rise to discrete and integral quantities.

For some very nice pictures of the standing waves that constitute atomic shells I recommend W. J. Wiswesser's 'The Periodic System and Atomic Structure', *Journal of Chemical Education*, July 1945, pages 312 and 318. The pictures may have been the origin of this theory, for I will never forget the thrill I experienced on seeing them and realizing how beautifully atoms could be explained and visualized in terms of waves.

Present quantum mechanics, however, as a whole, is unsatisfactory with respect to permitting one to visualize atomic and nuclear domains. One soon leaves three dimensions. Also one is confronted with the baffling wave-particle duality. The field theory on the

other hand permits complete and precise visualization of the field, and at any magnification. All that happens if the magnification is too great is that the field in the element of volume becomes uniform and nothing of interest is seen.

The chemical bonding conditions, which are fairly well understood, lead to a new level of stability and unity in the field, the level of molecules. Beyond the molecular level a new stabilizing and unifying condition on the ψ-field arises which is poorly understood, the life condition, which organizes the field into living cells. Beyond the life condition arises the living organism condition or conditions which permit many cells to form a new unity, the living organism. Beyond the level of the organism arises another level, still less well understood, the level of consciousness. With conscious individuals in existence various new levels arise permitting individuals to combine to form new unities such as human society and its institutions. And it seems likely that higher levels presently unknown and perhaps completely unsuspected may be possible and will in time evolve or be constructed.

At the other and simpler end of the scale there is some condition, still obscure, which permits the field to organize into elementary particles like the electron. I suspect that the condition has something to do with spin. If a standing wave of finite size is to be stable then, thinking of it as the sum of a forward wave from the left and a backward wave from the right, at the left edge of the standing wave the forward wave must start to build up where the backward wave was finally died, and everywhere left of the center of the standing wave the forward wave must be gaining energy by non-linear interaction from the backward wave. At the center however the direction of energy flow must reverse so that to the right of the center the forward wave gives up energy to the growing backward wave and finally dies out at the right edge of the standing wave. The fact that the standing wave must have a center at which there is a sign change in direction of energy flow, suggests that some vectorial pattern of ψ-distribution is rotating about the center so there is a vector reversal at the center, like the tangent to a wheel. I think of this rotating ψ-field pattern as the spin of the particle.

One of the most interesting and challenging areas for research is to try to discover the various conditions, like the integral quan-

tum number condition, which, by placing some restriction on the field equation, select from all its possible solutions a certain class which then constitutes a level in the hierarchy of entities derived from the field, such as elementary particles, atoms, molecules, living organisms, conscious beings, societies, and higher levels not yet known.

The Wave Unity in Place of the Wave-Particle Duality

In present quantum mechanics the generally accepted doctrine is that of the Wave-Particle Duality, according to which a particle like an electron can appear as a wave or as a corpuscular point particle but not as both at the same time. It is a peculiar and unsatisfactory idea but it is forced upon us by linear quantum theory. Consider electron diffraction by a crystal. To be diffracted by a crystal an electron must be a wave. But the electrons and nuclei in the crystal which do the diffracting cannot also be waves, at least as described by a linear equation such as Schroedinger's or Dirac's, because a linear wave cannot diffract a linear wave. Two linear waves don't interact. Neither knows the other is present. So the electron being diffracted is a wave but the electron or nucleus in the crystal doing the diffracting is a corpuscular particle.

With any linear wave equation such as Maxwell's, Schroedinger's, or Dirac's, it is quite impossible to account for the universe as a collection of waves and waves *only*. A linear wave does not interact with another linear wave: it cannot emit one or absorb one or reflect one or diffract one. If all atoms were linear wave-packets they would simply go their independent ways, and emission, absorption, reflection, and diffraction would not exist. If waves are linear then corpuscular particles or material bodies of some kind which are not linear waves must exist to emit, absorb, reflect, and diffract the waves. But if an electron is sometimes the linear wave that is dif-

fracted and sometimes is the non-linear corpuscular body that does the diffracting, then we are driven to the Wave-Particle Duality.

The situation is quite different if the wave equation is non-linear, as is the field equation. Then waves can and do interact, and a wave can absorb, emit, reflect, or diffract another wave. There is no longer any necessity to postulate the Wave-Particle Duality. We postulate the Wave Unity: the universe consists of a collection of waves and of waves only.

The qualitative picture of electron diffraction presented by the Wave Unity is that of an electron which is a small, stable wave-packet of some geometry, such as a spherical harmonic with probably a rotational wave which is its spin and which gives the packet its stability and a right- or left-handedness opposite to a positron's. The electron approaches the diffracting crystal because its forward wave component is greater than its backward wave component. It is an oscillation at a stable limit cycle in a non-linear field. It interacts with the standing waves that constitute the crystal and is diffracted, in which condition it is an expanding wave at an unstable limit cycle. If it were the only wave in the universe, that is if the same volume of space did not also contain a sum of other waves which we may call 'background noise', the expanding wave which is the diffracted electron would go on expanding forever even though unstable. The electron wave and the background noise waves interact non-linearly to upset the perfect balance of an unstable limit cycle, which then converges to the stable limit cycle which is a small stable wave-packet like the original electron.

If one knew precisely what the background noise was one could predict, using the field equation, exactly where the diffracted electron would nucleate out of the field as a small stable packet, much the way a water drop nucleates out of an unstable cloud of supersaturated water vapor. But one does not know what the background noise is since it is an extremely complicated sum of all the radiation pouring into the volume from the rest of the universe. One would have to know the state of the rest of the universe. Furthermore one cannot measure a field without changing it, so one cannot hope to measure the background noise. Therefore, while the whole process is determinate, as governed by the field equation and the rest of the universe, our knowledge of the process is incomplete

and must always remain so. For this reason the process appears to be governed by probabilities. If one repeats the experiment with a second electron under conditions which superficially appear to be identical, the background noise is in fact quite different and the stable electron condenses out of the unstable diffracted electron wave at a different point.

If the background noise is completely unknown about all we can do is to assume that it is uniform over the volume occupied by the diffracted wave. In this case the best we can say is that the probability that the small stable electron will condense and appear at any given point is proportional to the scalar of the matter quaternion $\frac{1}{2}\psi^{*}\iota\psi$, or $\frac{1}{2}(H^2+E^2)$, which is the energy-mass-charge-probability density of the diffracted wave. Hence we can interpret the complex electromagnetic wave $\psi=H+\iota E$ as the probability amplitude wave, and the matter quaternion $\frac{1}{2}\psi^{*}\iota\psi$ as the probability quaternion.

The wave unity clears up another unsatisfactory aspect of present quantum mechanics. The wave equations, like Dirac's or Schroedinger's, with usual Hamiltonians, are linear in ψ, and, once set up, do not involve the charge ρ. On the other hand ρ is given by a non-linear expression in ψ like $-\iota\rho+\rho v=\frac{1}{2}\psi^{*}\iota\psi$. Waves, as described by the wave equation, are linear in ψ, but corpuscles, as described by the probability for ρ, are non-linear in ψ. Non-linear interactions of ψ can affect ρ, but ρ cannot or at least need not affect ψ. If one were clever enough to set up Schroedinger's equation for the universe as a whole, it would predict how ψ would evolve thenceforth without involving ρ or any non-linearities. Thus the question of how linear or non-linear the universe is becomes very subjective with the individual scientist setting up his equations, and depends on whether and how often he elects to use the non-linear expression for ρ. The non-linear wave unity clears this up by incorporating the non-linear expression for ρ in the field equation, so one has no choice about using a linear equation for ψ or a non-linear equation for ρ.

Universal Causation, and its Reconciliation with Local Indeterminacy

ACCORDING to the field theory the only fundamental reality is the field vector ψ, and this vector is completely determined by the field equation. The universe is therefore completely causal.

The universe however is only causal as a whole. No isolated part of it constitutes a closed causal system by itself. The field in a finite volume of space is determined by the field equation operating on a set of initial and boundary values of ψ. The boundary values of ψ are not fixed constants nor are they variables which are determined exclusively by the given volume of space. The boundary values of ψ are the simultaneous solution of the field equation inside the volume and outside the volume. Therefore only the totality of volumes of space, which may be infinite in extent, constitutes a closed causal system.

From a different point of view, the ψ-wave at any point interacts non-linearly with, and therefore depends on, the ψ-waves arriving at that point from all the other points in the universe, or at least all the other points within the relativistic light-cone. Therefore all points within the light-cone are interdependent, and since all light-cones for sufficiently prolonged times become interdependent, all points in the universe become interdependent.

Within a finite volume all events are interdependent because the field is non-linear. The principle of superposition does not apply. It is not correct to select and segregate one solution of the field equation, say for one atom, in a container of finite volume, and neglect the solutions for all the other atoms in the container. All the solutions within a given volume interact non-linearly and modify each other.

We shall presently postulate that the field is truly unified and therefore constitutes the mental or spiritual universe as well as the physical universe. If this is true it is not correct to assume that the physical universe is a closed causal system exclusive of mental universe. Only both together constitute a closed causal system.

If only the universe as a whole is a closed causal system then

any part of the universe less than the whole does not constitute one and therefore is by itself indeterminate. Local indeterminacy is a necessary consequence of the fact that the universe is only causal as a whole.

For example a diffracted electron does not constitute a closed causal system and hence is by itself indeterminate, so one cannot predict where a single diffracted electron will appear. It is only the combination of the ψ-field of the diffracted electron, plus the ψ-field of the background noise in the same volume pouring in from all the rest of the universe, that constitutes a closed causal system and gives *the* total solution of the non-linear field equation for that volume at that time.

The background noise cannot be measured without being changed. In fact no field can be measured without being changed. In classical physics the universe was supposed to consist of particles whose positions and velocities completely specified the universe and its fields, which were created by the particles acting as sources. It was believed that in principle the position and velocity of a particle could be measured without affecting either, so in principle the complete specifications of the universe could be measured without altering the universe.

In a universe which consists of a field and only a field the universe is specified by the field vector at each point of space and time. Even a finite volume requires an infinite number of specifications, unlike the finite number of particle positions and velocities required classically. The slightest change in the field vector at any point changes the field as a whole. The slightest change caused by introducing the most delicate measuring instrument changes the field to be measured. And the field, to be measured completely, would have to be measured at every point, which would change it completely.

We see that we cannot hope to measure the field of the universe without changing it, and we cannot hope to determine the field completely for any part of the universe without knowing the whole field of the universe. Hence our knowledge of the universe is necessarily probabilistic or incomplete.

'Probabilistic' or 'chance' means 'incompletely known'. It does *not* mean 'causeless', that is, 'occurring for no reason whatsoever'. The outcome of the throw of a coin is probabilistic or chance

because a limited number of possible effects, heads or tails, is dependent upon an enormous number of causes, so complex they are not completely known. If we knew the initial position of the coin, the prevailing wind, the muscular tension and fatigue of the hand, the coefficient of friction of the thumb, and a host of other factors, we could predict the fall of the coin. The fall is causal but complicated, hence unknown, hence probabilistic.

A probabilistic universe is in no way incompatible with a causal universe. There is a widespread idea, which I think is a fallacy, that if events, particularly atomic events, occur probabilistically they must be causeless and happen for no reason at all. The idea is that two identical diffracted electrons will appear in two different places and so must be non-causal, since identical causes should produce identical effects. The fallacy lies in presuming the two electrons are identical. In linear quantum mechanics they are. In non-linear field theory they are not. Each electron interacts with the field background noise it finds, and the background noise from the rest of the universe is so complex the chances are negligible of its ever repeating itself. Only the combination of electron and background noise constitutes a closed causal system, and in general one does not find identical combinations twice in succession.

The whole idea of causelessness is, I think, utterly repugnant to science. Causelessness means that events occur for absolutely no reason at all. The first premise of science is that events have causes, and the whole object of science is to discover them. Furthermore the idea of causelessness intolerably complicates science, for if some events are causal, as they certainly are, and others are non-causal, then some super-causation must exist to determine which is which. One falls into an utter quagmire of confusion trying to conceive of a principle of super-causation that will determine when an event is to be non-causal, and how big an event will be permitted to be non-causal.

One of the strongest pieces of evidence that the non-linear field theory is correct is the simple way it simultaneously accounts for universal causation and local indeterminacy.

THE Uncertainty Principle states that the product $\Delta\omega\Delta t$ for a wave, thought of as a function of time, or $\Delta\omega\Delta x$ for a wave, thought of as a function of space, is a finite quantity of the order of Planck's constant h. By using the central idea of the Laplace or Fourier transform that frequency ω is equivalent to the differentiation operator $\partial/\partial t$ or $\partial/\partial x$, and the fundamental postulate of quantum mechanics that $\partial/\partial t$ and $\partial/\partial x$ can be replaced by energy and momentum, one concludes that $\Delta\varepsilon\Delta t = h$ and $\Delta p\Delta x = h$. In words, one cannot be exactly certain of the energy and time of measurement of a particle simultaneously, nor of its momentum and position simultaneously.

I think that the Principle is probably correct, but I think it has been misinterpreted to imply that the universe is non-causal.

In the first place the importance and fundamentalness of corpuscular particles has been exaggerated, due to the historical idea that particles are small billiard balls. In the field theory particles as billiard balls or point particles do not exist except as mental fictions for convenience. Particles are waves or collections of waves in the ψ-field. As we have already shown, point particles are only defined probabilistically by the ψ-waves since the ψ-waves defining a single particle interact non-linearly with the entire ψ-field of the universe, so the single particle is not completely known with certainty unless we know the complete field of the universe, which is impossible. Thus point particles are already probabilistic or uncertain or incompletely known, without reference to the Uncertainty Principle.

If one considers the Uncertainty Principle with regard only to ψ-waves, the fundamental reality, and not with regard to point particles, it seems to me a better name would be the Granularity Principle. The Principle states that if one makes a Fourier analysis of the ψ-waves which compose a stable particle or wave-packet, one is not entitled to use an infinite or continuous Fourier integral but must use a finite or discrete Fourier sum. In a Fourier integral $\Delta\omega\Delta t$ (or $\Delta\omega\Delta x$) approaches zero, whereas in a Fourier sum it remains finite.

A Fourier sum or harmonic analysis consists of specifying a time function by a finite number, say r, of ordinates, each at a certain time t (or location x for a space wave), or by specifying r amplitudes, each at a certain frequency ω.

1)

$$f(t_j)=\sum_{k=0}^{r-1} g(\omega_k)\frac{e^{\iota\omega_k t_j}\Delta\omega}{\sqrt{2\pi}}$$

2)

$$g(\omega_k)=\sum_{j=0}^{r-1} f(t_j)\frac{e^{-\iota\omega_k t_j}\Delta t}{\sqrt{2\pi}}$$

where

$$j=\frac{t-t_o}{\Delta t} \text{ and } k=\frac{\omega-\omega_o}{\Delta\omega}$$

The above equations are only valid providing that $\Delta\omega\Delta t=2\pi/r$, which is the classical or mathematical uncertainty principle and has nothing to do with physics.

The proof depends on the orthogonality with respect to summation of the exponentials.

$$S=\sum_{j=0}^{r-1} e^{\iota\omega_k t_j} e^{-\iota\omega_m t_j}$$

$$=\sum_{j=0}^{r-1} e^{\iota n\Delta\omega(t_o+j\Delta t)} \qquad \text{if } \omega_k-\omega_m=n\Delta\omega \text{ and } t_j=t_o+j\Delta t$$

$$=C\sum_{j=0}^{r-1} z^j \qquad \text{if } z=e^{\iota n\Delta\omega\Delta t} \text{ and } C=e^{\iota n\Delta\omega t_o}$$

$$=C\frac{1-z^r}{1-z}$$

$$=r\delta_{km} \qquad \text{if } \Delta\omega\Delta t=\frac{2\pi}{r}$$

$$=r \qquad \text{if } k=m$$

$$=0 \qquad \text{if } k\neq m$$

If one lets $\Delta\omega\to 0$, $\Delta t\to 0$, and $r\to\infty$ the finite Fourier sums become Fourier integrals:

$$f(t)=\int_{-\infty}^{\infty} g(\omega)\frac{e^{\iota\omega t}dw}{\sqrt{2\pi}}$$

$$g(\omega)=\int_{-\infty}^{\infty} f(t)\frac{e^{-\iota\omega t}dt}{\sqrt{2\pi}}$$

The point of the Uncertainty Principle is that physically, in describing a stable or nearly stable system of waves such as a rest mass particle, one is not entitled to let $\Delta\omega\to 0$ and $\Delta t\to 0$. One must describe the system by sums of frequencies which are multiples of a fundamental frequency, or sums of time intervals which are multiples of a fundamental period, or sums of space intervals which are multiples of a fundamental wavelength.

I interpret this as a Granularity Principle related to the fact that a non-linear differential equation typically exhibits a set of discrete stable solutions or limit cycles, each with its own frequency, wavelength, and period, or multiples of these, and it seems one can only describe the solution in terms of these discrete frequencies, periods, and wavelengths. The Uncertainty Principle thus becomes part of a more general principle that all the stable solutions of the non-linear field exhibit a discrete granularity including the fact that charges differ by the unit charge, aggregates of matter are incremented and decremented by discrete atomic amounts, quanta of emitted and absorbed energy are discrete multiples of Planck's constant, and stable particles must be described by discrete Fourier sums rather than continuous Fourier integrals.

I think the granularity probably only applies to stable or nearly stable solutions or limit cycles. I see no reason why an unstable solution which has momentarily diverged from a limit cycle cannot be described by an infinite Fourier integral.

In any case I see no necessity, because of the Uncertainty Principle, to postulate a lack of causality.

The Philosophical Hypothesis that the Universe is Unified, and the Original Derivation of the Field Equation Therefrom

I WAS first led to the field equation by the following line of thought. My point of departure was a scientific hypothesis or artistic feeling or religious faith that the universe is both unified and simple. Physics has reduced it to fields and elementary particles, but this is dualistic, not unified. Fields can exist, as empty space, without particles, but particles cannot exist without space, which is a field, in which to exist, so fields are more fundamental than particles. A unified theory of physics therefore should attempt to explain particles in terms of a field.

What kind of a field? The electromagnetic field is the best understood, so I decided to try to explain particles in terms of it. I dismissed the gravitational field as being of a much smaller order of magnitude, and therefore probably a second-order effect. I dismissed probability amplitude fields and nuclear fields as being too poorly understood. In any case, if the theory were to be successful, it must unify all these fields and show that they are essentially a single field under different circumstances, so it should be possible to derive the theory starting from any one of them.

Thinking of an electromagnetic field in terms of classical light waves, that is, solutions of Maxwell's equations, I postulated that *particles are standing light waves.*

A dilemma arose at once. Maxwell's equations are linear and so will not account for a stable standing wave of finite size in the absence of material boundaries. A standing wave consists of a sum of a forward and a backward wave, which, if linear, do not interact with each other, and so, if of finite extent, eventually pass each other, destroying the standing wave.

Maxwell's equations must be wrong. They must be a linear approximation to an exact non-linear equation. Furthermore Maxwell's equations are not self-sufficient since they involve the charge ρ. To form a self-sufficient system of equations one needs Maxwell's equations, the electromagnetic force law, and the mechanical laws of motion of the charged particles supposed to be the sources of ρ.

Finally all these laws break down in the atomic and nuclear domains. Could one solve all these difficulties by postulating that charge ρ is a non-linear function of the field variables E and H? This would non-linearize Maxwell's equations, make them self-sufficient by eliminating ρ, and since they would be self-sufficient they would have to be universally accurate or be fallacious.

What non-linear function of E and H might ρ be? The best known non-linear function of E and H is the electromagnetic field energy density $\frac{1}{2}(H^2+E^2)$ so I postulated $\rho=f(H^2+E^2)$ but I didn't know the function f. I tried various possibilities in terms of L. Silberstein's complex quaternionic formulation of Maxwell's equations, which I have always liked for its simplicity, compactness, and natural asymmetry. Silberstein defines the complex electromagnetic vectors $L=H-\iota E$ and $R=H+\iota E$, and gives Maxwell's equations as $\bar{D}L=\bar{C}=+\iota\rho+\rho v$. He gives the energy quaternion $\frac{1}{2}\iota L^*L=-\frac{1}{2}\iota(H^2+E^2)-E\times H$, and this struck a chord because it is well known that current ρv and Poynting flux $E\times H$ can be used interchangeably, so I tried $\bar{D}L=\pm\frac{1}{2}\iota L^*L$. This was not relativistically invariant but I found that $\bar{D}L=\pm\frac{1}{2}L^*\iota L$ is, if ι is transformed too.

Equating charge ρ, which can be positive or negative, to energy $\frac{1}{2}(H^2+E^2)$, which can only be positive, seemed hopeless, until I realized that changing the sign of L, which changed handedness, changed the relative signs of the linear term $\bar{D}L$ and the quadratic term $\frac{1}{2}L^*\iota L$, or the relative signs of $\bar{D}L$ and ρ in Maxwell's equations. At this point the theory really started to look hopeful, for here was a good explanation of the signs of charge, and of matter and anti-matter, and an indication that the equation applied in the domain of the elementary particles.

The form L^*L suggested $\psi^*\psi$, and I began to wonder if the complex electromagnetic vector could be identified with the probability amplitude vector ψ in Dirac's equation, which qualitatively seemed right. The idea proved fruitful and by a slight redefinition of the operator $\bar{D}$ gave Maxwell's equations as $D\psi=-\iota\rho+\rho v$, Dirac's equation as $D\psi=K\psi$, Dirac's charge $-\iota\rho+\rho v$ as $\frac{1}{2}\psi^*\iota\psi$, and the field equation as $D\psi=\frac{1}{2}\psi^*\iota\psi$.

An Hypothesis that a Creation-Annihilation Reaction Exists which is not Subject to the Second Law of Thermodynamics; or that the Universe is Eternal without Beginning or End

A WELL-KNOWN PROCESS is pair production, or the creation of two particles having rest mass, one of matter and the other of anti-matter such as an electron and a positron, from a single photon of electromagnetic radiation. So is pair annihilation in which an electron and a positron annihilate each other to produce two photons.

Pair production from a single photon is not possible in empty space since momentum would not be conserved, and so occurs near a heavy nucleus. So far as I am aware the possibility of *two* photons reacting in empty space to produce an electron-positron pair has not been postulated, possibly because photons are supposed to be linear waves which do not interact. In the field theory electromagnetic waves or photons are non-linear and do interact.

It is hard to avoid postulating that two electromagnetic waves can react to produce two rest mass particles since the essence of the field theory is that rest mass particles consist of standing electromagnetic waves or pairs of forward and backward waves locked together by their non-linear interactions. What could be more natural to the theory than that two independent electromagnetic waves under suitable conditions can lock together to form rest mass particles, and that under other conditions the rest mass particles can separate into independent electromagnetic waves?

We postulate therefore that a creation reaction exists which is the reverse of the observed annihilation reaction and consists of *two* photons or electromagnetic waves reacting in empty space to produce two particles having rest mass, one of matter and the other of anti-matter. More generally we postulate that both the creation and annihilation reactions constitute a single creation-annihilation reaction which may be driven in either direction according to LeChatelier's Principle, depending on the relative concentrations of rest mass particles and electromagnetic waves.

With such a reaction we can explain the energy balance of the

universe. In the stars the annihilation reaction predominates while in empty interstellar space the creation reaction predominates. The stars convert rest mass into electromagnetic radiation which is propagated into interstellar space, where the creation reaction reconverts it to rest mass which gravitates back to the stars. In this cycle the energy of the universe, including rest mass energy, is conserved, as required by the general conservation law implicit in the field equation.

It is obvious however that while such a cycle obeys the conservation law or the first law of thermodynamics it violates the second law of thermodynamics.

But it is also obvious that the second law of thermodynamics cannot be correct for the universe as a whole for all times past and present. The second law states that an isolated system always goes from a state of temperature differences to a state of temperature uniformities, or a state of available energy to a state of unavailable energy, or a state of order to a state of disorder, or a less probable state to a more probable state. While the law is undoubtedly true for some domain, it cannot be true universally for all places and all times, past and present. For consider the paradox of the creation of the universe. Either the universe had a beginning or it did not. If it had no beginning but has been in existence for an infinite time, then the second law says that it should be completely run down and at a uniform temperature. It is not. So either it had a beginning or the second law is false. But if it had a beginning the second law is also false, for the act of creation surely violated the law. To make this clear suppose that the creation of the universe occurred in two stages: first a metaphysical stage which created the ψ-field out of nothing (whatever that means) but left the value of ψ zero at every point, and second a physical stage which lifted the zero ψ-field to its past and present non-zero values. The physical stage clearly violated both the first and second laws of thermodynamics since it created energy where none existed and it organized this energy from a uniform null field into temperature differences and available energy. So in any case the second law is not universally true.

The blunt fact of the matter is that the universe is demonstrably a perpetual motion machine, the Patent Office to the contrary notwithstanding. If it was created at some finite time in the past it is

a perpetual motion machine of both the first and second kinds, violating both the first and second laws of thermodynamics. If it was not created but has existed forever it is still a perpetual motion machine, but only of the second kind. Since the field equation embodies the first law of thermodynamics, an act of creation, which would violate the first law, would destroy the field equation. Therefore we conclude that the universe had no beginning but has existed forever and will exist eternally.

If the second law is not valid universally we must ask in what domain it is valid, for it is certainly valid in some domain, as any engineer who ignores it while building a steam power plant will find out. We postulate that the second law is valid in any region composed only of matter, or in any region composed only of anti-matter, but that it is not valid in any region composed equally of matter and anti-matter. If this is true it is not valid for the universe as a whole, which is composed equally of matter and anti-matter, nor for the creation-annihilation reaction, which involves equal amounts of matter and anti-matter, but it is valid on earth, which is composed of matter. The creation-annihilation reaction then becomes a 'Maxwell's demon' or mechanism by which the second law is circumvented and an eternal universe explained.

According to this hypothesis the primary source of energy of the stars is the annihilation reaction, which converts rest mass energy into electromagnetic radiation. In interstellar space the creation reaction reconverts the radiation back into rest mass which gravitates back to the stars or gravitates together to form new stars. The process is truly reversible and lossless, and so is not subject to the second law. Hence the cycle repeats endlessly.

Other primary sources of energy for the stars have been postulated, including combustion, gravitational shrinkage, nuclear fission, and nuclear fusion. All depend upon a consumable and irreplaceable fuel, and hence cannot explain either a universe without a beginning which would have consumed all fuel supplies by now, nor the creation of a universe that had a beginning since a consumable and irreplaceable fuel could not have supplied the original store of itself.

If the universe did indeed consist of matter, without anti-matter, then the annihilation reaction would not be available and one would

have no choice but to postulate that fission or fusion was the primary energy source. It is unlikely that any creation reaction exists that will produce matter alone without anti-matter, or anti-matter alone without matter, from electromagnetic radiation, since it would imply that the universe is preferentially right- or left-handed. There would be no process then for reconverting electromagnetic radiation into new fissionable or fusionable matter. The stars would run out of fuel when they had converted all their original fuel into nuclear ash, or elements in the middle of the periodic table, and, as predicted by the second law which would apply, the universe would come to a dead uniform temperature.

If another part of the universe consisted only of anti-matter, without matter, it too would come to a dead uniform temperature. If however the two dead parts of the universe, one of matter and the other of anti-matter, now drifted into each other and intermingled, the universe would come to life again with a bang and the annihilation reaction would convert all the dead matter and anti-matter into electromagnetic radiation, which is generally conceded to be an available form of energy. We see that the annihilation reaction is indeed an exception to the second law and would be capable of regenerating a universe of equal parts of matter and anti-matter, if the parts separately ran down under the operation of the second law.

In discussing the general conservation law derived from the field equation we noted that the term E·J, which in Poynting's theorem is the Joule heating loss, becomes E·E×H, which is identically zero. This may be interpreted to mean that, at the elemental field level, the basic processes, such as the creation-annihilation reaction, are lossless and completely reversible. There is no Joule heating loss because there is nothing to heat. Losses and irreversibility imply that there are alternate channels into which energy can flow. Typically we try to convert chemical energy into mechanical energy and lose some to heat. At the elemental field level there is neither chemical energy, mechanical energy, nor heat. In the creation-annihilation reaction all energy involved must either be electromagnetic, i.e. travelling wave energy, or rest mass, i.e. standing wave energy, and, since energy is conserved, if it is not one it must be the other. There are no alternatives, hence no losses, hence no irreversibility, hence no second law.

The creation reaction explains Olbers' paradox that the night sky is dark. Olbers observed that if stars are evenly distributed in space the number of them for a given increment of distance from the earth goes up as the square of the distance, while the light received on earth from a given star goes down as the square of the distance, so equal increments of distance should produce equal increments of light. An unlimited universe, in which an infinite number of increments of distance exist should produce an infinite amount of starlight in the night sky. But the night sky is dark. The explanation is that the creation reaction in interstellar space absorbs as much energy as the stars produce by the annihilation reaction, so the average electromagnetic energy in space is a constant, and is the amount necessary to maintain the creation reaction in space in equilibrium under LeChatelier's Principle. If it dropped, creation reactions would become less likely and the electromagnetic energy in space, with a new supply constantly pouring in from the stars, would start to rise. If it rose, the reverse would occur.

The same equilibrium promises to explain why a certain proportion of the energy of the universe is in the rest mass of the stars and the balance is in the electromagnetic radiation in space; and why therefore the stars occupy a certain proportion of the total space of the universe.

A similar equilibrium promises to explain how large are the alternate regions of matter and of anti-matter, since too small regions would be unstable due to too frequent collisions or mixings with consequent annihilations, while too large regions would be too difficult to form since the chance of an anti-matter particle travelling a very long distance through a region of matter, or vice versa, is slight.

Such collisions and consequent annihilations as do occur may explain some of the releases of high energy observed by astronomers. Cosmic rays can be accounted for as the result of the annihilation of anti-matter particles produced in space by the creation reaction in a region of matter. Supernovae can be accounted for as the result of the collision of sizable bodies of anti-matter drifting into a region of matter or vice versa.

Still another equilibrium appears to exist. Presumably the creation reaction usually produces electron-positron pairs but occasionally a proton-antiproton pair, if the photons are of sufficient

energy. In a region of matter like the solar system the anti-matter particles either escape from the region or are destroyed by the annihilation reaction. The electrons accumulate on the sun and planets, giving them negative charges which repel further electrons but attract the rarer protons, until an equilibrium is reached where neutral matter survives, consisting of equal numbers of electrons and protons. I understand such a negative charge has been measured on the surface of the earth.

We have already postulated that the universe is not expanding but that the astronomical red shift is due to the non-linear interaction of electromagnetic waves in space. The fact that the shift is always to a lower frequency indicates that energy has been lost from the observed wave to space. Where does the lost energy go? We suggest into the creation reaction.

To summarize, we postulate that a creation-annihilation reaction exists which is not subject to the second law of thermodynamics, and that the reaction is the primary and continuing source of energy in the stars and of rest mass in space.

The universe is non-expanding, conservative, self-renewing and eternal, without beginning and without end.

Consciousness

CONSCIOUSNESS, our inner awareness of existence, is the basic data of science and the most valued treasure of each human being. As such it must be reckoned with in any unified theory that claims to encompass existence. It remains mysterious as ever, but if the field theory is correct some logical deductions can be made concerning its nature and properties.

We ask whether existence is truly unified in a single entity, the field, or whether it is dualistic in the sense that Descartes and many others have postulated, and consists of two entities, mind and

matter, or spirit and matter. We discard Descartes' dualism for the reason that the field, which has been developed to describe matter, much more nearly fits Descartes' definition of mind or spirit than his definition of matter. Physics has been dematerialized. The ψ-field is completely characterized and described by the properties of a complete vacuum. The classical or primitive definition of the spiritual is that which can exist in empty space free of matter. Descartes' definition of spirit or mind was that it was a non-material entity without extension in space. While the ψ-vector through its square accounts for matter it is not itself material, and while its components H and ιE exist in space they do not have extension in space: it is not correct to think of them as spatial vectors or arrows with their tails at one point and their heads at another. The H and ιE vectors at a point exist entirely and only at that point, without spatial extension, and at any other point entirely different vectors exist. It is better to think of them as pinpoint discs of color which can face in any direction and so have x, y, and z components, and which have magnitudes or intensities of color at each point, and which are of two colors like red and yellow which are orthogonal (neither contains any of the other) to each other in a two-dimensional non-extensive color space, like 'red'+ι'yellow'. It is perhaps better still to think of them as abstract and essentially unexplained entities having certain simple postulated properties in terms of which we hope to explain all the complexities of experience.

The field therefore fulfills both the classical definition of the spiritual and Descartes' definition of the mind. It is Descartes' matter, the fundamental entity of classical physics which like ivory filled billiard ball atoms, that has been discarded. It would be redundant to postulate two entities, the mind and the field, or the spirit and the field. The spirit is the field. The mind is the field. We postulate that the field is truly unified and encompasses all of existence: matter, mind, and spirit. If the field is truly unified we are at liberty to call it the physical universe or the spiritual universe or both.

We do not attempt to understand what the ultimate nature of consciousness is, any more than we attempt to understand what the ultimate nature of the quantities H and E is. With Kant, we believe it is impossible to understand 'the thing in itself'. Understanding

consists of explaining the complex in terms of the simple, the unfamiliar in terms of the familiar. We cannot understand the simple and the familiar: we can only accept them. Understanding is like the mathematicians' asymptotic series, which give better and better answers for a while as one pushes farther out into the series, but then get worse and worse if one pushes too far. Philosophers often push their trade too far by trying to explain the simple, and end up being obscure.

There are at least four things about the field which are experimental facts but which we do not understand and, at least for the present, do not try to understand: why the field exists, why it is quaternionic, why it obeys the field equation, and why it is capable of being conscious. But, given these, we do attempt to explain the rest of existence, physical, mental, and spiritual, in terms of the field.

One conclusion we come to in the field theory is that potential consciousness is a universal property of the field everywhere, including the field in a complete vacuum. We admit that consciousness can move around in the field, since we admit that other humans are conscious like ourselves, and to us they move. We have to be a little careful here because, due to relativity, we do not admit that we ourselves move with respect to the field, or at least we cannot detect our own uniform motion. Now nothing is transported in the classical mechanical sense when a person moves: he and his consciousness are propagated as a wave to a new region of the field. The new region becomes conscious and the old region that the person left subsides into unconsciousness. Thus any part of the field including a complete vacuum can be stimulated into consciousness in a very short period of time by having a suitably organized standing ψ-wave, such as a human brain, propagated into it. We conclude that potential consciousness is a universal property of the field.

A second conclusion we come to is that, just as a field theory denies the existence of a corpuscular billiard-ball atom which can be tagged and numbered, and its individual history followed from now to eternity, so the theory denies the existence of a spiritual or mental corpuscular atom which can be tagged and followed individually to eternity. All that exists in a field is waves, and while

waves can be identified individually and followed for a while under favorable circumstances, they cannot be permanently tagged and followed individually through such processes as diffraction and absorption. There are no permanent partitions in a field, physically or spiritually. The field is a unity, a single whole. A standing wave which is an atom may endure as a recognizable individual for a very long time, and so may a standing wave which is a human consciousness, but the division from the rest of the field is not permanent, and eventually the standing wave will revert to the field as a whole.

A third conclusion is that memory is the thread of continuity which constitutes an individual personality, or what we mean by the first person singular 'I'. Consider a victim of amnesia, who loses his memory while retaining his consciousness. The victim, to himself, becomes a new and different person, a new 'I', unrelated to the former inhabitant of his body. Without memory, the standing wave which is a human brain and consciousness, would in propagating through space become a new and different individual at every instant, without continuity of individual personality. It is memory, itself a standing wave, which relates and associates a consciousness at one moment of time and in one region of space with a sequence of consciousnesses at preceding times and other regions.

We will put these three conclusions together in the next chapter.

Finally, we ask ourselves whether consciousness is an entity that comes into being at some level of organization of the field, perhaps only in humans or more probably in many of the animals; or is consciousness an ever-present entity at all levels but one which gradually fades as one descends from the human through the animal kingdom to the vegetable kingdom and the inanimate world. In the traditional language of religion, the question we ask is whether there is such an entity as the human soul. Anyone who has played a game against a digital computing machine has come face to face with the Frankenstein problem, or the problem of the existence of the human consciousness or soul. By means of a typewriter the machine and the human player converse in plain English and the impression to the human is overpowering that he is conversing with another conscious being like himself. But is he? I am extremely skeptical. If we admit that an electronic computer is conscious, then any relay or vacuum tube must be conscious, and we become

Animists believing that rocks have eyes and shells have ears. No, I think a computer is just an unconscious assembly of inanimate parts, a subconscious intelligence, a simulator of the human consciousness but not the real thing. If so, then the human consciousness or soul is an entity that is not present at all levels of the field but comes into being at one of the higher levels through some as yet mysterious unifying and stabilizing principle similar to but more subtle than the integral quantum number principle. The integral quantum number principle is a constraint upon the field equation which permits an atomic shell to come into existence, and the shell is an entity which unifies and stabilizes the field throughout the sum of an infinite number of differential elements of volume. The atomic shell is something new and different from the sum of its parts. So I think that consciousness is some unifying constraint on the field equation that permits a collection of atoms constituting a brain to become a new entity, an individual personality or soul, which is greater than and different from the sum of its parts. I doubt if the digital computer manufacturers have yet succeeded in creating a consciousness or soul.

The Death of a Wave and the Immortality of the Field

An ocean wave makes up out at sea, rolls in on the beach, and breaks. The individual is born, lives, dies, and is gone forever. But the ocean is immortal. It lives on and experiences wave after wave.

The human consciousness is a wave in the field. It is born, lives, and dies. But the field is immortal, and experiences wave after wave.

Is each of us the wave which dies and disappears forever, or are we the immortal field? Both, I suggest. It seems to me the answer depends entirely on one's point of view.

If I adopt the point of view that 'I' am the wave, then 'I' am a

particular thread of memory, a specific standing wave configuration of a brain shaped by my particular personal history. With death, memory almost certainly disappears, since it seems extremely likely that it depends on a particular configuration of brain cells and atoms in the brain cells. To envisage memory continuing without the brain is like trying to picture the smile on the Cheshire cat without the cat. I think that memory and the personal individuality that depends on memory, the thing we mean by the pronoun 'I', is the wave and disappears forever when the wave breaks.

But if we adopt the point of view that 'we' are the field, meaning by 'we' all the consciousnesses that collectively make up the subjective universe, then 'we' are immortal and experience consciousness after consciousness, as new waves make up in the field. This is not the doctrine of reincarnation. Reincarnation postulates corpuscular spiritual atoms which at least in principle can be tagged and followed individually from body to body to eternity, a postulate which the field theory denies. The wave making up out at sea is not the reincarnation of the wave breaking on the beach. 'I' am not the reincarnation of John Jones or Julius Caesar or any previous 'I'. All I say is that 'we', the conscious field, are the same field of which John Jones and Julius Caesar were parts.

All I postulate is that conscious existence consists of the repetition of what each of us has already experienced: the birth of a new consciousness and a new memory, not traceable individually to any previous consciousness or memory; life, death; the birth of a new consciousness and a new memory, not traceable to any previous; life, death; and so on. This seems to me a modest and scientific postulate, since it is based only on actual experience and the premise that what has demonstrably happened once can happen again.

This volume is not concerned with human values and desires but I cannot refrain from expressing the opinion that if we are immortal in the sense I suggest, it is the most attractive kind of immortality I can imagine. Many people think they want to retain their memories eternally, but I think they haven't thought through what such a hideous fate would be like. Nothing would or could ever be quite new again. Even in middle age, life begins to lose the entrancing lustre of newness it had in youth, and by the end of the first million years, not to say the first zillion, most of us would be

intolerably bored with ourselves. The only way we can be new again is to lose memory. Memory, with the individual personality, the 'I', that accompanies it, is our most valued possession, but it is also a prison wall which prohibits us from being anybody else or anybody new. We, the immortal field, wish to escape the prison wall from time to time and enjoy being somebody else and somebody new.

Free Will in a Causal Universe

THE unified field theory implies that universal causation exists, the field being causally determined by the field equation and its prior state. We have postulated that the field is truly unified and embraces mental and spiritual existence as well as physical existence.

We conclude that free will exists.

Free will means that alternatives exist in our minds from which we are free to choose one, and having chosen we can impress our will on the physical universe and alter it. Alternative possibilities consist of solutions of the field equation for hypothetical initial and boundary conditions. An enormously large multiple infinity of such solutions exist. The process of choosing consists of following a logical and therefore causal chain of reasoning or perhaps an illogical but still causal chain of prejudice, instinct, or error. Having chosen we take action and change the physical world which in the unified theory is not a separate universe from the mental and does not constitute a closed causal system by itself. It is only the complete universe, physical and mental as a whole, that constitutes a closed causal system.

Before history unfolds, many hypothetical future histories are possible, as defined above, which we may entertain in our minds before choosing one, but we know that a single unique history of the universe will unfold, and after it has done so we can look back and see that it was the result of a unique causal chain, if we include

all the mental links. As participants we are responsible for making rational choices based on good cause to provide the mental links, for if we were to abandon that responsibility and become fatalistic observers the physical universe by itself would follow a totally different course of history.

Or, to quote, 'We are free to do as we like although we are not free to like as we like'.

I feel that some philosophers and physicists have leapt at the concept of a non-causal universe in an attempt to justify the existence of free will. Their argument, which I think is fallacious, seems to be as follows: the universe is dualistic, and consists of the physical universe and the separate mental universe. If the physical universe is completely causal by itself without reference to the mental, then mental activity cannot affect the physical world and we do not have free will. They say that only if the physical world is non-causal can we have free will. They believe in free will. Therefore they postulate that the world is non-causal.

The fallacy is thinking that the physical universe is separate from the mental. The reason the physical universe is not separately causal from the mental is not that it is non-causal but that it is not separate.

They oppose the fatalists who argue that the universe is dualistic, that the physical universe *is* causal without reference to the mental, and so it doesn't matter what we think or will because our minds cannot change the physical world. The fallacy again is thinking that the physical universe is separate from the mental.

In contrast to both I argue that the universe is unified and consists of a non-material ψ-field which is capable when suitably organized of forming both matter and mind. A train of thought which is a changing ψ-field is also a train of physical activity and can affect the physical world. The physical universe is not separate from the mental world, nor separately causal without reference to mental activity. If the physical world were non-causal then a well-considered train of thought could not be relied upon to produce the physical action desired, for a gap in the chain of cause and effect between thought and action could produce arbitrary or contrary action. In short a non-causal world would not be a free world but an insane world.

The Implications of the Field Theory: Evolution to the Unknown

If the universe is causal, as determined by the field equation, one may ask whether the future can be predicted and the implications of the field equation elucidated in complete detail. The answer is that the future is unknown and will remain so. For humans to calculate mathematically the future history of the universe by means of the field equation is utterly impossible. The initial conditions of the universe are unknown. They cannot be measured without being changed. If known the future could not be computed on any computing machine which was not as large and complete as the universe itself. In fact, mathematically, the universe *is* the computing machine that determines the future from the past. No smaller computer would suffice. All that humans can hope to predict mathematically from the field equation are approximate answers for fictitious and simplified initial and boundary conditions, answers having limited accuracy and good only for limited regions and for limited periods of time.

One can however postulate and perhaps prove that certain principles are implied by the field equation. For example we have proved that energy is conserved. A very broad and basic principle which appears to be implied by the field equation is the principle of evolution: what can survive does survive. This may be only a truism, but it has caused a revolution in human thought in the last century and promises to continue to be of central importance. The principle applies not only in the area of organic evolution, but also in the inanimate. Stable atoms survive radioactive atoms. Stable combinations of right- and left-handed field configurations, such as electron-protons, survive unstable ones, such as electron-positrons. Separated regions of matter and anti-matter survive mixtures. An interesting speculation is whether the field equation itself is the product of the struggle for survival, in competition with other possible equations. For example the field equation embodies the conservation law. Another equation which did not might fail to survive because its field would collapse or blow up, while the conservative

field maintained itself stably. Is the field equation the only possible fundamental law, or can man produce artificial conditions under which another field equation applies, just as he produces artificial elements which do not exist naturally?

Existence may be looked upon as a gigantic and many-pronged experiment of Nature's to find out what can survive. At first when a new star or galaxy is born, by an automatic process of trial and error, of natural variation and selection, level after level of new entities build up in the field, each having some new principle of stability which permits it to endure: elementary particle, atom, molecule, living cell, living organism, consciousness, human or similar being, society, and probably higher levels yet to come, such as individuals with enlarged consciousnesses and faculties, or a society with a common consciousness into which its individual members can plug themselves through nerve fibers strung around the countryside in place of telephone wires. At the sub-human animal level, with the communications and intelligence necessary for co-operation still unevolved, the means of survival has largely been tooth and claw. At the human level, with speech and recorded knowledge and foresight available, tooth and claw are probably no longer the appropriate means of survival and may well prove suicidal. Co-operation and non-lethal forms of conflict, such as the conflict of ideas, probably become the new means appropriate to the new conditions. Furthermore, with conscious intelligence available, evolution, particularly of human societies and perhaps of human faculties, is no longer limited to blind trial and error but may be directed by conscious foresight. Man and his ideas evolve to an unknown future. Men postulate as scientific hypotheses or accept as revealed truths what they think the future should be and strive to make it that. And history records which men's progeny and which men's ideas survive.

One may ask why, if the field has existed forever without any beginning, the process of evolution isn't exhausted. Why haven't all the possibilities been tried?

A possibility is a solution of the field equation for hypothetical initial and boundary conditions. It does not violate the equation but it may violate the actual conditions. The number of possibilities which exist is a multiple infinity of a vastly larger order of magni-

tude than the single infinity of actual configurations of the field that have occurred in an infinite length of time. The number of events which have occurred remains and will always remain a vanishingly small proportion of the events which might have occurred. Evolution always remains near the beginning of its possibilities, even though it has endured and will endure forever.

Life is always new and the possibilities are always boundless. We, mankind, to the best of our knowledge the advance guard of evolution, have the power within our grasp to select and control the possibilities for our continued survival and evolution, and to maintain and improve our position as the advance guard. Yet even if mankind fails and is swept into oblivion as just another of evolution's many negative experiments, we, the eternal field, from consciousness to consciousness, whether in man, his competitors, or his successors, sweep on to a new and boundless future.

Science and Religion

In the unified field theory it is postulated that the universe consists of a field, probably eternal in time and infinite in extent, which is governed by the field equation. From the elemental field the operation of the field equation creates and evolves the elementary particles, atoms, molecules, and living and conscious organisms which constitute existence. Since the field equation is self-sufficient and completely determines the field it brooks no rivals. The field equation is the fundamental law and the only fundamental law of existence, and rules the universe.

The first article of faith of the monotheistic religions is that there is a God, there is only one God, and God rules the universe.

The field theory either confirms or denies, gives support to or is in conflict with, the monotheistic religions.

If one defines the word 'God' to mean a Being external to the

universe, or an anthropomorphic Being constituting only a small part of the universe, or a Being who obeys no law, not even His own, or a non-existent Being invented for malevolent purposes of oppression, then the field theory and religion are in conflict.

Webster's Unabridged Dictionary defines the word 'God' to mean 'The Supreme Being; the eternal and infinite Spirit, Creator, and Sovereign of the universe'.

If one accepts Webster's definition of the word 'God', then the field theory and the monotheistic religions are in essential agreement. The universe is a field, and the field equation is God, the Supreme Sovereign of the universe.

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